DESIGN OF THE STRUCTURE OF THE COMPOSITE PACKET IN OPTIMUM THIN-WALLED STRUCTURAL MEMBERS

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Greater interest in the composite materials caused by their high specific characteristics has resulted in an increase in the number of investigations in the area of technology and mechanics of these new materials. The largest possible set of the components of the composites and the large variety of methods of formation of the structure and of technology organization make it possible to apply real control parameters but require, at the same time, the application of a special approach to realize these advantages on the level of completed products. This approach must be based on reliable methods of analysis and assumes the application of the methods of optimal design and experiment planning taking into account the technical purpose of structures, various service limitations, and possibilities of technological realization. The formulation and solution of the problems of designing optimum structural members are both of independent interest and also make it possible, by applying the decomposition methods [1, 2], to interpolate the resultant organization principles in a specific iteration method in designing large systems.

In this article, we examine the problems of determining the optimum structure of the packet of thin-walled sheets and shells made of single layers of composite materials. We examine the so-called "maximum perfection" structures, i.e., designs which optimize certain characteristics determining the performance of members, i.e., strength, stability, stiffness, etc., at a fixed mass and limitations imposed on other parameters. The controlling variables, the reinforcement angle \( \phi (\alpha_3) \), reinforcement coefficient \( \theta (\alpha_3) \) are represented in the form of required functions in the thickness of the packet (\( \alpha_3 \) is the direction along which the characteristic dimension is considerably smaller than the two remaining dimensions \( \alpha_1, \alpha_2 \)). As a result of the application of the iteration procedures using the method of spline functions, the infinite-dimensional problems are reduced to a sequence of finite-dimensional problems with the target functionals of the minimax type. Problems of this type are solved using packets of programs whose algorithms were adapted to take into account the polymodality which is typical of the examined group of problems. The proposed representation of the controlling parameters in the form of required functions makes it possible to determine most conveniently also the rates of variation of the parameters in the case of small dimensions. This may prove to be important in a number of problems, especially for local quality criteria.

The resultant data are compared with the characteristics of the optimal layered structures which are the result of a partial case of the examined approach. This also makes it possible to evaluate the additional gain which may be obtained as a result of variable compositions.

The thin-walled members of structures made of composites can be calculated in a comparatively wide range of the geometrical parameters and external loads using calculation models based on the conventional hypothesis of the type of the Kirchhoff-Love hypothesis of the theory of layered thin-walled anisotropic bodies [3]. The advantages of these models are both the fact that they have been developed to a sufficient extent and that methods are already available for evaluating the accuracy of the model. This is of special importance for structural members made of composites because of a number of their specific properties (flexibility, low shear characteristics, etc.).

The typical feature of design of structures of composite material is the presence, in the base set of the design space, of parameters associated with the structure of the packet of the material in addition to traditional [4] geometrical (functions of thickness, position of junctions of rod structures, form of the median surface), mechanical (elasticity moduli, density), and other control parameters. For example, for a given technological system, the single layer can be determined using the characteristics of the reinforcement and binder and the quantity representing the density of reinforcement in the single layer. The matrix of the elastic constants in the main physical axes of the single layer permits the representation

\[
\bar{B}_n = B_0 (\bar{B}_n, \bar{B}_o),
\]

where \( \bar{B}_n, \bar{B}_o \) are the matrices of the elastic constants of the reinforcement and the binder.
As a result of the gradual deposition of the single layers, we obtain a packet which is either anisotropic (for arbitrary skew layup) or its characteristics are sufficiently similar to those of the orthotropic material (crossed layup) where the elementary layer whose main physical axes form specific angle $\varphi$ with the geometrical axes of the structure alternates with the identical layer with the angle $-\varphi$, and a number of these pairs of single layers $N_j$ in the elementary packet is sufficiently large [5-7]. For these packets, the matrix of the elastic constants may be represented by the expansion

$$B(\varphi, \theta) = \sum_{k=0}^{3} b_{k}(\varphi, \theta) u_{k}^{1}(\varphi) u_{k}^{2}(\varphi).$$

Here $u_{1} = \cos 2\varphi (\alpha_{3})$, $u_{2} = \sin 2\varphi (\alpha_{3})$; matrix $b_{00}$ characterizes a specific corrected transversely isotropic material and has the form

$$b_{00} = \begin{pmatrix} B_{11}^{*}, & B_{12}^{*}, & 0 \\ B_{12}^{*}, & B_{11}^{*}, & 0 \\ 0, & 0, & B_{16}^{*} \end{pmatrix}.$$

The matrix coefficients

$$\begin{array}{l}
b_{10} = B_{0}^{-} \begin{pmatrix} 1, & 0, & 0 \\ 0, & -1, & 0 \\ 0, & 0, & 0 \end{pmatrix}; \quad b_{20} = G_{0}^{-} \begin{pmatrix} 1, & -1, & 0 \\ -1, & 1, & 0 \\ 0, & 0, & -1 \end{pmatrix}; \quad b_{02} = b_{12} = b_{21} = 0; \\
b_{01} = -\frac{1}{2} B_{0}^{-} \begin{pmatrix} 0, & 0, & 1 \\ 0, & 0, & 1 \\ 1, & 1, & 0 \end{pmatrix}; \quad b_{11} = G_{0}^{-} \begin{pmatrix} 0, & 0, & -1 \\ 0, & 0, & 1 \\ -1, & 1, & 0 \end{pmatrix};
\end{array}$$

determine the contribution of anisotropy, and

$$\begin{array}{l}
B_{0}^{-} = 0.25 (B_{11}^{*} - B_{22}^{*}); \quad G_{0}^{-} = 0.25 (B_{11}^{*} + B_{22}^{*} - 2B_{12}^{*} - 4B_{16}^{*}); \\
B_{11}^{*} = 0.25 (B_{11}^{*} + B_{22}^{*} + 2B_{12}^{*} + 4B_{16}^{*}); \\
B_{12}^{*} = 0.25 (B_{11}^{*} + B_{22}^{*} + 2B_{12}^{*} - 4B_{16}^{*}); \\
B_{16}^{*} = 0.25 (B_{11}^{*} + B_{11}^{*} + B_{22}^{*} + 2B_{16}^{*}),
\end{array}$$

and quantities $B_{ij}^{*}(\theta)$ are the corresponding constants of the single layer.

For each layer in this formulation we obtain the vector of the controlled dimensionality parameters $2N_j$; in the case of high values of $N_j$ typical of the majority of members of structures working in extreme conditions this causes principal problems in applying numerical algorithms. In addition to this, in translating the design into practice it is necessary to know all the specific features of the optimum regulations, taking into account the special features of technology. Consequently, it is convenient at high $N_j$ to change from the splines of the zero order with single defect [8] $S_{91}(\Delta N_{j})$ and uniform grid $\Delta N_{j}$ for the j-th layer which are used for the controlling functions $\varphi(\alpha_{3})$, $\theta(\alpha_{3})$ (the direction along which the characteristic dimension is considerably lower than the two remaining directions $\alpha_{1}$, $\alpha_{2}$) to splines of a higher order and evaluate the nature of optimal designs even for a small design space. Thus, in subsequent considerations we shall assume $(\varphi, \theta) \in S(n, j)$.

It should be mentioned that optimum designs with small jumps of the stress state are realized in the presence of defects $\nu \leq n$. Taking into account Eq. (1), it may easily be seen that in the given formulation the strain state in the general case is determined by the functional

$$\begin{array}{l}
\varepsilon_{xij}(\varphi, \theta) = \int \varepsilon_{xij} u_{1} h_{1} r_{x} \alpha_{3} d\alpha_{3},
\end{array}$$

The stress state will be determined both by the functional and functions $\varphi(\alpha_{3})$, $\theta(\alpha_{3})$. The controlling parameters may also include the quantities $I = (l_{1}, l_{2}, \ldots, l_{n})$, $H = (h_{1}, \ldots, h_{n})$; these quantities are the vectors of identification of the material and vectors of the relative thicknesses, where $l_{s} = j$ indicates that the s-th layer counted from a specific fixed reference surface in the conditioned direction is occupied by the j-th material, and $h_{s}$ indicates the thickness of this layer.

The members of the structure and, consequently, the structure of the packet as a whole may be evaluated using certain functionals reflecting specific qualitative features. The integral criteria include mass, critical forces, frequency of natural oscillations, integral stiffness. All these criteria will be determined by the quantities $\beta$, $I$, $H$. 

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