NONLOCAL THEORY OF STRUCTURED COMPOSITE MATERIALS*

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The axiomatic method of development, which is a familiar classic, makes it possible to construct the mechanics of a deformable solid (MDS) without internal contradiction, accurately, and on well-established grounds, being based on systems of calibration experiments on objects that can be described by theory. In pure mathematics, axioms of calibration experiments are, of course, absent, because the latter do not require them. In MDS, the position is such that calibration axioms should be introduced into the circle of axioms of this science.

The phenomena of inelasticity, plasticity, and viscosity are dynamic by their own nature; in essence, therefore, the axiomatics of deformation mechanics should also contain dynamic calibration. Attempts at static calibration are known and are internally contradictory, and, obviously, it is therefore precisely these that are unproductive: until now, the plasticity problem, especially the dynamic problem, has been urgent by nature, and awaits a correct realistic mathematical description within the framework of contemporary formalism permitted by advanced mathematics (theory of differential equations in partial high-order derivatives, or let us say, functional analysis).

The mechanics of the American school (Hull et al.) included axioms of thermodynamics among the MDS axioms, noting the familiar direction of the situation (in determining the basic concepts of thermodynamics, for example, entropy, there are so many contradictory elements that the calorie concept, which in many respects resembles the Carnot–Trusdell entropy (1972), is used in lieu of this concept in the American version of the system of axioms).

In the axiomatics that we propose, thermodynamic axioms are present in a form where there are 16 equally correct formalisms, which are, in principle, equally permissible for use; of these, we can state a preference for Planck’s formalism. The equally correct presence of all possible formalisms of the thermodynamic description of nonequilibrium systems makes our axiomatics nonminimal, but enables us to correlate this description with the descriptions accepted in related scientific disciplines (for example, with the thermodynamics of biosystems, Prigoshin et al.).

It is frequently accepted to close MDS equations (conservation of mass, impulse, and energy) by annexing to them equations of state (taken in the form of functionals of the $\sigma - \varepsilon$ relationship) without coupling equations (Maxwell and others); we prefer, however, to incorporate visibly coupling axioms among the MDS axioms, denouncing violation of chemical–bonding axioms as a definition of the concept of composite media. As far as we are concerned, therefore, MDS equations are closed by coupling equations (after Maxwell— with the exception of the chemical bonding; this brings us to the nonequilibrium and metastable nature of composites), and are calibrated experimentally in dynamics.

Since the coupling equations are solved (are satisfactory in the theoretical–functional sense), this leads us to realization of the classical notion of the adequacy of mechanical calibration of equations of state at the modern mathematical level. This calibration has become the center of theoretical constructions and a procedure for the metrological software that is required in practice. We can envision the possible use of functional–power series for the mathematically correct entry of $\sigma - \varepsilon$ relationships within the framework of the causality principle, describing the multiplicity of channels for the realization of nonequilibrium processes. We incorporate this context into the principle of addition of cooperative effects; in contrast to the prevailing interpretation for which the formalism stipulates only the lag of effects with time, we consider the possibility of interactions, which is also "spread" throughout space.

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In examining the space-time deformable continuum of the theory of elasticity, the processes in which are reversible, every nonlocal interaction takes into account within the framework of the mathematical formalism itself not only the direct interactions, but also interactions of events via intermediate points-bodies, which are "insufficiently” distant in some sense from the point of development or the point of realization of the effects. As is the custom in thermodynamics, this formalism does not presume that in effect, we know in detail the internal structure of the processes taking place in the channels of dynamic interaction of the remaining points-bodies. The formalism of the theory is phenomenological; it permits mathematical description of phenomena of nonlocal interactions of a different nature, requiring only one description — that the constants and influence functions permit physical measurability, as this is needed for dynamic calibration of the theory within the framework of metrological support.

The thrust of the calibration principle requires that theoreticians involved with development of mathematical descriptions for the nonlocal character and delay of plastic and other irreversible variations in material give special attention to possible realization of the mechanism of energy dissipation, but does not absolutely assume detailed (let us say, quantum-mechanical, or, more broadly, issuing from "primary principles") knowledge: dynamic calibration of the theory is sufficient in experiments. As practice has shown, a familiar flexibility of intellect is required of specialists in order to understand this: in the modern stage of development of the theory, singular functions on the basis of which we can lay out the operators of $\sigma-\varepsilon$ functional relationships in conformity with the coupling principle are introduced in MDS. The coupling principle makes it possible to reduce the matter to a problem that can be described mathematically correctly in differential equations in partial high-order derivatives. Thus, we arrive at principal transcendental theories that can be classed according to the nature of the equations obtained in this case. All theories known up to now (including, for example, hypoelasticity, which was introduced by the American MDS school) follow from this.

We separately isolated the description of a continuum, the influence functions of which are close to the fundamental functions of Bateman's equations (normally a telegraph equation, but this term is not very appropriate in MDS context), and treated it as an example of a nonlocal description. Within the framework of this approach, the effects in the composite are described by new types of $\sigma-\varepsilon$ relationships, the nonlocal character of which suggests the promise of the possibility of describing the properties of composites as an interacting matrix and medium — a skeleton that permits anisotropy, and an amorphous binder — within the framework of a formalism that is sufficiently suitable from positions of mathematics. The nonlocal character of the operators makes it possible to describe discontinuous fields of physical properties within the limits of "spread" influence functions for interactions that are, by their own nature, dynamic. It is possible to discern a specific analogy between this approach, which is based on nonlocal interactions, and the conventional theory of polycrystalline media, where an isotropic body, which is sufficient for practice, usually serves as an accurate analogy of the polycrystal under investigation; even the closeness of this replacement of a real polycrystal by an isotropic body is frequency forgotten, and the accuracy of the description justifies this.

Thus, a system of new representations, the detailed description of which (within the framework of the familiar mathematical formalism of classical functional analysis) becomes an MDS problem, like the problem of the theory of differential equations in higher derivatives, which has been expanded vigorously both theoretically and also practically in computational mathematics, is assumed within the framework of MDS axiomatics.

The theory proposed below contains thermodynamic formalisms, equations of mechanics, coupling equations (mechanical, chemical, and thermal processes), dynamic calibration, equations of nonlocal $\sigma-\varepsilon$ relations of the principle of the addition of cooperative effects, and the formalism of the coupling principle and principal theories that generate different systems of equations of mechanics in practice.

The mechanics of composite media reduces the problem itself to integrodifferential equations, which are usually closed by equations of state [1, 2], operating with potentials that are functionals of functions of required smoothness of coordinates and time in accordance with fundamental principles of thermodynamics and chemistry, and mainly with energy $E(S, e_{ij}, n_k)$ with the variation $\delta E = T \delta S - \sigma_{ij} \delta e_{ij} + \mu_k \delta n_k$, where $S$ is the Carnot entropy and $\mu_k$ are chemical potentials, and exergy $EX(S, e_{ij}, \mu_k) = E - \Omega$, where the chemical potential $\Omega$ is determined with enthalpy $H(S, e_{ij}, n_k) = E - \Omega$, where the elastic potential $\Omega = e_{ij} e_{ij}$, and exergy $HX(S, e_{ij}, \mu_k) = EX - \Omega$, with free energy $F(T, e_{ij}, n_k) = E - \Xi$, where the thermal potential $\Xi = TS$, and free exergy $FX(T, e_{ij}, \mu_k) = EX - \Xi$, with free enthalpy $G(T, e_{ij}, n_k) = H - \Xi$ and free enthalpy $GX(T, e_{ij}, \mu_k) = HX - \Xi$; in this case, the free enthalpy $G$ is exhausted by the chemical potential $\Phi$ according to Gibbs, and $G = \Phi + GX$ is possible when $GX = 0$ according to Planck.

There also exists the Carnot–Massey–Planck formalism (see [3]), which is based on use of Carnot entropy $S_0 = S(E, e_{ij}, n_k) = \frac{E}{T}$ with variation $\delta S = \frac{1}{T} \delta E - \frac{\sigma_{ij}}{T} \delta e_{ij} - \frac{\mu_k}{T} \delta n_k$, Carnot entropy $S_1 = SX(E, e_{ij}, \mu_k) = S + \Phi / T$, entropy $S_2 = K(E, e_{ij}/T, n_k) = S + \Omega / T$, entropy $S_3 = KX(E, e_{ij}/T, \mu_k/T) = S + \Omega / T$. 

289