Structures made out of composite materials possess a number of distinctive features, namely, nonuniformity of the material of the structure and nonuniformity of its stress state, anisotropy of the physicomechanical and thermophysical properties, and a strong dependence of the physicomechanical properties of the material on temperature. In addition for structural reasons the reinforcement angle can be variable along the length of the shell, and the structure of the layer stack can be asymmetric. All of these distinctive features should be taken into account in the calculation of the stress-strain state.

One of the main questions in connection with the solution of these problems is the choice of a model of the material. As is well known, structures made out of composite materials are heterogeneous structures formed by a combination of reinforcing elements (fillers) and an isotropic adhesive (matrix). The variety of kinds of fillers (glass, carbon, boron, and other kinds of fibers) as well as methods of combining them, i.e., the fabrication technologies (continuous winding, pressing, etc.), required the development of numerous models of the material [1-10].

The main aim of these theories is the establishment of the physical relationships between the stresses and strains in terms of the structural parameters of a material (the physicomechanical characteristics of a structural element, the reinforcement coefficient, and so on); the choice of the structural element (or the level of the treatment) has the widest range—from the elements of the filaments and the matrix [1, 2] to the stack element as a whole [4]. The methods of solution of this problem are diverse—from the principle of energy smoothing [1] to the solution of the elasticity theory problem of a system of doubly periodic nature [2]. We note that the choice of the model of a material predetermines the appearance of the samples with which the physicomechanical characteristics of the material (filler and matrix, unidirectional samples, tubular samples, model shells, and so on) are determined and is an important stage in the calculation.

The next stage in the solution of the problem formulated is the determination of the stress-strain state. References [9-14] are devoted to the solution of problems of the statics of laminated shells. In a number of them [12, 13] the solution is carried out by the scheme of network analysis (without taking account of the adhesive). The problem of the determination of the stress-strain state of a laminated anisotropic shell with constant elasticity parameters has been solved in [11]. Solutions of a number of problems for nonuniform (with variables in the coordinates but single-layered) bodies of revolution (in particular, for a hollow cylinder) are obtained in [14] by the methods of elasticity theory. For a symmetric stack structure the general solution in the displacements for a zero-moment laminated cylindrical shell with constant elasticity parameters has been obtained in [9, 10]. All the solutions listed have been obtained without taking account of the effect of temperature (and consequently the temperature dependence of the physicomechanical properties).

Thermal problems are discussed in [11, 15, 16]. Thus the solving equations for an orthotropic shell of revolution are obtained in [11, 15] with account taken of the temperature dependence of the physicomechanical properties of the material. However, one should note that the solving equations are obtained with a number of additional assumptions (the law of variation of temperature with thickness is assumed to be linear, the thermal expansion coefficients also depend linearly on temperature, etc.). Numerical methods of calculating the stress-strain state of laminated shells of revolution with variable elasticity parameters (including thermal problems) have been developed in [17-19].

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1. Computation Model of the Shell Material and the Basic Hypotheses. In constructing a computational model of a structural material produced by continuous winding, one can proceed from three basic assumptions. In the first approximation reinforcing fillers and adhesive are taken as the basic elements [2]. In the second case an elementary strip impregnated with a polymerized adhesive is taken as the basic element [9, 10]. And finally, in the third case the material of the entire stack of the shell is assumed to be the basic element [4].

As has already been noted, the choice of the basic element of the computational model of a structural material predetermines the type of samples necessary for obtaining the elastic and strength characteristics of the material which are subsequently used as the initial data for calculation of the structure. In the first case reinforcing fillers and the adhesive stand out as the samples, in the second case—unidirectional annular samples, and in the third case—samples cut directly out of the structure or tubular samples simulating its reinforcement scheme.

It is necessary in connection with the use of the first model for determination of the elastic and strength properties of a structure to apply theoretical dependences [2]. However, the existing theoretical dependences for determination of elastic and strength characteristics do not take account of a number of engineering factors, for example, the tension of the tape during winding, the heat treatment regime, and so on which affect the mechanical properties [10]. The use of the third approach is complicated for two reasons. First of all, the fabrication of samples made out of the structure leads to the effect of cut-off fibers [20], which does not permit reliably determining the mechanical characteristics of the material in the structure. Secondly, and this complication is a fundamental one, it is necessary in the determination of the mechanical characteristics with tubular samples simulating the reinforcement scheme of the structure to consider all possible reinforcement schemes if we want to obtain the optimal structure of the stack. This leads to an indefinitely large amount of experimental investigations. Thus it does not appear possible to use the first and third approaches for a reliable estimate of the stress–strain state.

In view of the fact that the structure is formed by successive overlapping of elementary layers of reinforcing fillers impregnated with adhesive, an elementary strip in the form of a tape impregnated with a polymerized adhesive is taken as the basic element [9, 10]. The choice of such an element seems justified, since the shell is formed precisely from it in the winding process. The elementary orthotropic strip is provided with strength under tension–compression in two directions and under shear, and the corresponding elastic and strength constants are determined experimentally with unidirectional annular and flat samples.

By elementary layer (monolayer) we shall understand a cylindrical shell wound at an angle $\phi$ to the generatrix (Fig. 1) and impregnated with a polymerized adhesive. The reinforcement angle $\phi$ varies from layer to layer (perhaps as a continuous or discontinuous function of the coordinate $\gamma$), and it can also vary along the axial coordinate $\alpha$ within the confines of a single layer (in the case of nonlinear winding; see Fig. 1).

The following hypotheses are adopted: the shell material is assumed to be linearly elastic; the shell thickness $h$ is assumed to be small in comparison with the radius $R$ ($h/R = 1/20$); and the Duhamel–Neumann thermoelastic hypothesis.

2. Physical Relationships with Thermal Action Taken into Account. Following [9], we obtain relationships which relate the stresses to the strains in the original coordinate system in terms of the elastic characteristics of the material $E_1^i$, $E_2^i$, $\mu_{12}^i$, $\mu_{21}^i$, $G_{12}^i$ (see Fig. 1). The structure of the structural material (the reinforcement scheme) is determined by the reinforcement angles $\phi_i$ and the corresponding thicknesses of the layers $h_l$ ($l = 1, 2, 3, ..., k$).

We shall introduce a coordinate system $(\alpha_i, \beta_i)$ associated with the reinforcement direction of the $i$-th layer. The stresses for the $i$-th layer are expressed in the $(\alpha_i, \beta_i)$ system in terms of the stresses in the