EVALUATION OF THE SURFACE STRENGTH OF DISPERSION-HARDENED COMPOSITES SUBJECT TO PRELIMINARY ELECTROCHEMICAL TREATMENT

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The machining of composite materials of the type of dispersion-hardened hard alloys is associated with significant difficulties caused not only by their high hardness, but also by the significant nonuniformity and difference in the properties of the components. These same properties of a hard alloy can be used, however, as factors that facilitate treatment by using methods based on selective failure of the components. Methods of electrochemical diamond grinding (ECDG), which are based on the effect of a combination of mechanical cutting by diamond grains, electroerosion, and electrochemical solution, are placed in the same class.

To compute rational regimes and provide for the required surface quality, it is necessary to solve the problem concerning the variation in the surface strength of the composite after the treatment.

A layer of a composite material which is subjected to electroerosion treatment is discussed. This layer is modeled by a perforated plastic with elliptical holes having random axial lengths. The distribution of the lengths of the hole dimensions and the distances between them is determined experimentally. The distribution of structural elements of the composite and the porosity and mechanical properties of the components are also known. The problem is stated and solved: find the relative variation in the strength of a composite plate and establish the dependence of softening on the characteristics of the hole distribution, the mechanical properties of the components, and the structure of the composite.

The problem involving the deformation of an infinite plate of a two-phase material perforated with elliptical holes having semiaxes of random dimensions is solved. The plate is subjected to uniform tension of intensity \( \sigma \). At each point in the plate, there is a plane stressed state with stress-tensor components \( \sigma_1, \sigma_2, \) and \( \sigma_{12} \) which are random fields of the coordinates \( x_s, s = 1, 2 \).

At each point, let us formulate the failure criteria in the form \([1, 2]\)

\[
F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1; \quad i, j = 1, 2, 6,
\]

where

\[
F_1 = 1/X - 1/X_0; \quad F_2 = 1/Y - 1/Y_0; \quad F_6 = 1/S - 1/S_0;
\]

\[
F_{11} = 1/XX_0; \quad F_{22} = 1/YY_0; \quad F_{66} = 1/SS_0.
\]

Here \( X \) and \( Y \) are the ultimate tensile strengths in the longitudinal and transverse directions, \( S \) is the ultimate tensile strength in shear, and \( X_0, Y_0, \) and \( S_0 \) are the corresponding compressive strengths. The quantities \( F_i \)
Fig. 1. One of realizations of random process.

and $F_{ij}$ are random fields of the coordinates $x$. For a statistically isotropic medium, these random fields are transformed into random processes, one of the realizations of which is shown in Fig. 1.

Let us average Eq. (1) in terms of the density condition:

$$f(E^{(1)}, E^{(2)}, \sigma^{(1)}, \sigma^{(2)} | E),$$

where $\nu = 1, 2$. In conformity with methods of conventional moments [3], we then obtain a system of three algebraic equations in terms of conditional mathematical stress expectancies:

$$\sum_{i=1}^{2} \sum_{s=1}^{3} \{F_i \rho_{sv}(x) \sigma^{sv}(x) + F_{ij} \rho_{erv}(x) (\sigma^v)^r \} = 1,$$

where $\sigma^{sv}$ is the mathematical expectancy of the stress $\sigma_i$ at point $x^{(i)}$ with the condition that the component $s$ is found at point $x^{(i)}$, and the component $\nu$ at point $x^{(j)}$; $\rho_{sv}(x)$ is the probability that the component $s$ exists at point $x^{(i)}$ with the condition that the component $\nu$ is found at point $x^{(j)}$, and $F_i(s)$ and $F_{ij}(s)$ are strength parameters of the component $s$; in this case, it is assumed that inside the holes,

$$F_{ij}^{(s)} = F_{ij}^{(s)} = 0.$$  \(\text{(2)}\)

Representing $\sigma^{sv}(x)$ as the sum of regular and random components, we obtain

$$\sigma^{sv} = \langle \sigma^{sv} \rangle + \sigma^{sv}; \quad \langle \sigma^{sv} \rangle = \text{const.} \quad \text{(3)}$$

Let us substitute (3) in (2) and take into account the properties of the conditional probabilities $\rho_{sv}(x^{(j)})$ [3]:

$$\rho_{sv}(0) = \delta_{sv}; \quad \rho_{sv}(\infty) = C_v,$$  \(\text{(4)}\)

where $C_v$ is the concentration of the component.

When (3) and (4) are taken into account, we obtain, in place of (2), a strength criterion expressed in terms of the average stresses $\langle \sigma^{sv} \rangle$ in the components and the dispersions $D_{ij} \sigma^{(2)}$ of these stresses. It is assumed in this case that failure will occur in the matrix. For the matrix, the failure criterion can be written from (1) in the form

$$F_0^{(a)} \langle \sigma_1 \rangle + F_0^{(a)} \langle \sigma_2 \rangle + F_1^{(a)} \langle \sigma_1 \rangle + F_{12}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle = 1,$$ \(\text{(5)}\)

where $\langle \sigma_2 \rangle^2 = \langle \sigma_2 \rangle $, and the subscript 2 denotes the matrix. Equation (5) is an equation of the effective ultimate strength surface in the matrix, which is expressed in terms of average stresses.

Let us examine the case of macroscopic uniaxial tension-compression, when the average stresses in the matrix are $\langle \sigma_1 \rangle$, while $\langle \sigma_2 \rangle = \langle \sigma_2 \rangle = 0$. Equation (5) assumes the form

$$F_1^{(a)} \langle \sigma_1 \rangle + F_{12}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle + D_{22}^{(a)} \langle \sigma_2 \rangle = 1.$$  \(\text{(6)}\)

It is easy to demonstrate that this is a quadratic equation, the smaller root of which corresponds to the case of tension, and the larger to the case of compression. To determine the ultimate strength of the matrix, it is necessary to determine the stress dispersion $D_{ij} \sigma^{(2)}$ in the matrix in the presence of elliptical holes. Let us transform the coordinates $x_1$ and $x_2$ in the form $x_1 = x_1$ and $x_2 = a x_2/b$, where $a$ and $b$ are correlation scales of the geometric dimensions of the holes in the $x_1$ and $x_2$ directions, respectively. In this case, we have an isotropic field of circular openings with random dimensions in the transformed system of coordinates. In the transformed plane $(x_1, x_2)$, we obtain the stress dispersion in the form

$$D_{ij} = D_{EE} \langle \epsilon \rangle^2 + 2D_{EE} \langle E \rangle + D_{E} \langle E \rangle^2,$$ \(\text{(7)}\)