STRESS DISTRIBUTION IN MULTILAYERED COMPOSITE MATERIAL WITH CURVED STRUCTURES (MODEL OF A PIECEWISE HOMOGENEOUS BODY)*

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For the investigation of various problems of the strength, stability, and oscillations in connection with composite materials with curvatures in the structure it is indispensable to have information on the distribution of stresses and strains on areas whose dimensions are commensurable with or smaller than the dimensions of the curvatures. Existing publications (e.g., [1-6]) dealing with composites with curved structures applied principally continuous approaches, and the effect of the curvature in the structure was taken into account in the calculation of the derived mechanical characteristics.

On account of the generally known tenets, no continuous theory [1-6] of structurally inhomogeneous materials can in most cases provide reliable information of a quantitative and also of a qualitative nature on the distribution of stresses and strains in each component of a composite material; such information can be obtained only within the framework of a model of a piecewise homogeneous body with the application of a strict three-dimensional theory.

Articles [7-9] suggested an efficient method of investigating the state of stress and strain of laminated and fibrous composite materials with curved structures, and the method was developed for the model of a piecewise homogeneous medium and the application of the


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equations of the theory of elasticity. With the aid of this method the authors of [10, 11] investigated the effect of curvatures on the stress distribution in laminated and fibrous composite materials with low concentration of filler where in effect the mutual effect of adjacent curved layers of filler is not taken into account.

In the present work we apply the method of [7-9] to investigate the effect of curvatures in multilayered composite material on the stress distribution in the case of uniaxial loading in the direction of the lamination, and here the mutual effect of all layers of filler is fully taken into account. It is assumed that the material of the layers of the matrix and of the filler is homogeneous and isotropic.

1. Using the approach of [7-9], we investigate the stress distribution in composite material which has an infinite number of layers curved in single phase and alternating in the direction of the Ox 2 axis, with normal forces uniformly distributed "in infinity" which act with intensity \( \langle \rho \rangle \) in the direction of the Ox 1 axis. We note that \( \langle \rho \rangle \) is the stress averaged over the entire area of the body under consideration on which a normal external force acts in the direction of the Ox 1 axis.

Taking into account the periodicity along the Ox 2 axis with the period \( 2(H(z) + H^{(1)}) \) \( (2H(z) \) is the thickness of the layer of filler; \( 2H^{(1)} \) is the thickness of the layer of the matrix), we take from the layers under consideration the two layers \( 1(z) \) and \( 1^{(1)} \) (Fig. 1), and for them we carry out the entire procedure of the solution. Here we assume that between the layers of filler and matrix conditions of complete adhesion obtain. We take the equations of the central layer of filler in the form \( x_2 = \varepsilon f(x_1) \), where \( \varepsilon \) is a dimensionless small parameter \( (0 \leq \varepsilon < 1) \).

In accordance with [7-9] we seek the values of stresses, strains, and displacements in the form of series of the parameter \( \varepsilon \) in the form

\[
\sigma_{ij}(m) = \sum_{k=0}^{\infty} \varepsilon^k \sigma_{ij}^{(m)k}; \quad \varepsilon_{ij}(m) = \sum_{k=0}^{\infty} \varepsilon^k \varepsilon_{ij}^{(m)k}; \quad \varepsilon_{ij}^{(m)} = \sum_{k=0}^{\infty} \varepsilon^k \varepsilon_{ij}^{(m)k}.
\]

Here the generally accepted designations were used.

We note that in determining the zero approximation we find that the values of this approximation correspond to the state of stress of the composite material under consideration without curvature of the layers of filler with the specified form of action of external forces, and they are balanced with these forces:

\[
\sigma_{ij}^{(1)0} = \langle \rho \rangle \left( \eta^{(1)} + \eta^{(2)} \right) \frac{E^{(2)}}{E^{(1)}} \left[ \frac{1 - (\nu^{(1)})^2}{1 - (\nu^{(2)})^2} \right]^{-1}; \quad \eta^{(m)} = \frac{H^{(m)}}{H^{(0)} + H^{(1)}};
\]

\[
\sigma_{11}^{(2)0} = \frac{E^{(2)}}{E^{(1)}} \left[ 1 - (\nu^{(1)})^2 \right] \sigma_{11}^{(1)0}; \quad \sigma_{ij}^{(m)0} = \sigma_{22}^{(m)0} = 0; \quad m = 1, 2;
\]

here the magnitudes marked by the superscripts (1) and (2) relate to layers of matrix and to layers of filler, respectively; \( E^{(m)} \) is Young's modulus; \( \nu^{(m)} \) is the Poisson ratio. Therefore, the values of the first, second, and of subsequent approximations correspond to the local self-balanced state of stress induced by curvatures in the structure of the investigated composite material.