responding to the experimental values; however, since the calculated curves of the intensity of photoionization through the $T_1$ and $S_1$ levels in the nonlinear region are close together, the predominant channel of photoionization cannot be distinguished according to the nature of the experimental curves in a nonlinear region.

From the data cited it can be seen that the quantum yields of the two-step photoionization of organic molecules in solution through intermediate excited states are rather large. Thus, at densities of the exciting radiation $\sim 100 \text{ MW/cm}^2$, characteristic of the pumping of lasers based on solutions in organic compounds by monopulses of solid-state lasers and their harmonics, a substantial fraction of the active molecules are ionized.

**LITERATURE CITED**


**PHASE FILTER OF SPATIAL FREQUENCIES**

**WITH AN ASSIGNED ABSOLUTE VALUE OF THE PULSE RESPONSE**

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The main goal of coherent optical filtration is to transform the spectrum of the spatial frequencies of an image [1]. However, there may be a different goal, viz., to obtain a diffraction pattern with an a priori assigned amplitude distribution. The phase can then be arbitrary. If we wish to obtain the amplitude distribution $f(x, y)$ in the focal plane of an objective, a transparent filter with the amplitude transmission

$$T(\xi, \eta) = t(\xi, \eta) e^{i(\varphi(\xi, \eta))}$$

should be placed in front of the objective, and a parallel light beam should be directed at it along the axis. Here $t$ and $\varphi$ are the modulus and the argument of the complex number $T$; $\xi = kx_0/f'$ and $\eta = ky_0/f'$ are the spatial frequencies; $x_0$ and $y_0$ are the coordinates in the plane of the filter; $k = 2\pi/\lambda$ is the wave number; $f'$ is the focal length of the objective; $f \exp(i\psi)$ is the pulse response of the filter; $\psi$ is the arbitrary (in our case) phase of the diffraction pattern.

A significant shortcoming of filter (1) is the great loss of the energy of the radiation due to its amplitude part. This loss is greater, the greater is the region in which $f(x, y) \neq 0$. For this reason, the use of an arbitrary phase rule in order to replace a amplitude-phase filter (1) by a purely phase filter was suggested in [2]. The purpose of the present paper is to substantiate theoretically the method for the calculation of a phase filter, which was tested on the examples in [2].

A phase filter is a transparent plate of variable optical thickness. The phase shift created by $\varphi(\xi, \eta)$ is called the phase characteristic of the filter. Placing a rectangular filter with the aperture dimensions $2a \times 2b$


in front of the objective (better up against it) in a parallel beam of coherent light, we obtain the following dis-

tribution of the amplitude in the rear focal plane:

\[ g(x, y) = \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} e^{i(\varphi(x, y) - xz - y\eta)} \, dz \, d\eta, \quad (2) \]

where \( \alpha = k\alpha/l \) and \( \beta = k\beta/l \).

Our purpose is to find the phase characteristic of the filter \( \varphi(\xi, \eta) \) for which the amplitude distribution

\( g(x, y) \) created by it would be as similar as possible to the desired distribution \( f(x, y) \). We shall characterize

the degree of similarity by the correlation coefficient

\[ P = \frac{\left[ \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} f(x, y) \, g(x, y) \, dx \, dy \right]^2}{\int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} f^2(x, y) \, dx \, dy \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} g^2(x, y) \, dx \, dy}. \quad (3) \]

This parameter reaches its maximum value, which is equal to unity, only in cases in which \( g(x, y) \) is propor-
tional to \( f(x, y) \). The denominator in expression (3) for an assigned function \( f(x, y) \) is a constant, since by

virtue of Parseval's theorem we have

\[ \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} g^2(x, y) \, dx \, dy = 16\alpha^2\beta^2. \]

Thus, it is sufficient to maximize the functional

\[ H = \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} f(x, y) \, g(x, y) \, dx \, dy, \quad (4) \]

i.e., to find the function \( \varphi(\xi, \eta) \), which, when substituted into (2) and (4), would bring \( H \) to its maximum value.

This variational problem can be reduced to an integral equation by representing the integrals in (2) and (4) in

the form of sums and setting the derivative of the functional \( H \) with respect to a local value of \( \varphi(\xi, \eta) \) equal to

zero. With accuracy to the insignificant constant factors we have

\[ \sum_{m,n} f_{mn} g_{mn} = 16\alpha^2\beta^2. \quad (5) \]

where \( f_{mn} \) and \( g_{mn} \) are the values of the functions \( f(x, y) \) and \( g(x, y) \) at discrete points in the sampling \((x_m, y_n)\),

and, according to (2),

\[ g_{mn} = \sqrt{C_{mn}^2 + S_{mn}^2}, \]

\[ C_{mn} = \sum_{i,j} \cos (\varphi_{ij} - x_{m_i} - y_{n_j}), \quad S_{mn} = \sum_{i,j} \sin (\varphi_{ij} - x_{m_i} - y_{n_j}). \quad (6) \]

Here \( \varphi_{ij} \) is the value of the function sought \( \varphi(\xi, \eta) \) at the point \((\xi_i, \eta_j)\). The derivative of functional (5) with

respect to the local value \( \varphi_{kl} \) is equal to

\[ \frac{\partial H}{\partial \varphi_{kl}} = \sum_{m,n} f_{mn} \frac{\partial g_{mn}}{\partial \varphi_{kl}} = \sum_{m,n} f_{mn} \frac{g_{mn}}{g_{mn}} \left[ C_{mn} \cos (\varphi_{kl} - x_m - y_n) - S_{mn} \sin (\varphi_{kl} - x_m - y_n) \right]. \quad (8) \]

Making the transition from the sums back to the integrals in Eqs. (7) and (8) and discarding the indices,

we obtain an integral equation which expresses the condition of an extremum of functional (4)

\[ \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} \frac{f(x, y)}{g(x, y)} \left( \cos [\varphi(\xi, \eta) - x\xi - y\eta] S(x, y) - \sin [\varphi(\xi, \eta) - x\xi - y\eta] C(x, y) \right) \, dx \, dy = 0, \quad (9) \]

where \( C \) and \( S \) are the real and imaginary parts of the complex amplitude

\[ G(x, y) = \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} e^{i(\varphi(x, y) - x\xi - y\eta)} \, dx \, dy = g(x, y) e^{i\varphi(x, y)}. \quad (10) \]

created by the phase filter.

Equation (9) reduces to the form

\[ A(\xi, \eta) \sin \varphi(\xi, \eta) = B(\xi, \eta) \cos \varphi(\xi, \eta), \quad (11) \]