EFFECT OF PUMPING NONUNIFORMITY ON THE MODE STRUCTURE OF THE RADIATION FROM A SOLID-STATE LASER

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Repeated investigations into the pumping energy density distribution of solid-state lasers [1-4] reveal that the pumping is usually nonuniform over the cross section of the active elements. In this paper we shall study the influence of these nonuniformities on the mode composition of laser emission, as well as the generation thresholds of various modes and also their amplitudes. The computing method employed in this paper is largely similar to that indicated in [5], but differs from the latter in allowing for the thermal lens formed in the active element during generation. The resonator mirrors are considered as being plane-parallel (Fig. 1). A calculation may be carried out on an analogous basis for spherical mirrors and lenses specially introduced into the resonator.

During the calculation we shall determine the thresholds and amplitudes of the modes arriving at the state of generation as the pumping energy is successively increased. The basis of the calculation is the balance equation representing the relationship between amplification and losses for steady-state oscillations [5],

\[ n \int u_{jk} G_{jk} (1 - \Sigma A_{m} |u_m|^2) G_{js} = \frac{p_{jk}}{\rho} + \frac{\lambda_{jk}}{\rho}, \]

where \( n \) is the excess of the pumping intensity above the threshold value of the fundamental mode (which is taken as 1); \( A_{m} \) is the relative amplitude of the \( m \) type of oscillations; \( \lambda_{jk} \) and \( p_{jk} \) are the matrix coefficients of the diffraction losses \( \lambda \) and the losses due to incomplete reflection and inactive losses \( p \); \( G \) is a function describing the nonuniformity of the pumping distribution over the cross section; and \( u_{jk} \) are the eigenfunctions of a resonator incorporating a lens, which (according to [6]) take the form

\[ u_{jk} = c_{jk} H_j \left( \sqrt{\frac{\lambda}{\rho}} w \right) H_k \left( \sqrt{\frac{\lambda}{\rho}} w \right) e^{-\frac{i}{\rho} (x^2 + y^2)}. \]

Here \( H_j \) and \( H_k \) are Hermitian polynomials; \( \rho \) is the spot radius of the fundamental mode; and \( q \) is the variance at the lens site. Both \( w \) and \( q \) may be expressed in terms of the focal length of the lens \( f \) and the distance from the lens to the mirrors \( d \) by means of equations derived in the manner of [6]:

\[ q = -d + i [f (2 - d/f)]^{1/2}, \]

\[ w^2 = \frac{4f d}{k_0 [f (2 - d/f)]^{1/2}}. \]

The pumping distributions relative to the \( x \) and \( y \) axes were assumed to be mutually independent and were expressed in the form of expansions in Hermite polynomials

\[ G(x, y) = \sum_p a_p H_p (x) \sum_n b_n H_n (y) \]

and in the form of Gaussian curves

\[ G(x, y) = e^{-\frac{1}{2} (ax^2 + by^2)}. \]

The magnitudes of the pumping nonuniformities along the x and y axes were represented by the parameters $r_x$ and $r_y$, equal to the half-width of the function $G(x, y)$ at a level of $1/e$ of the maximum value measured along the corresponding coordinates. In the case of severe nonuniformities ($r_x \leq r_y \ll w$), $G(x, y)$ may be more conveniently described in the form of a Gaussian curve. The expansion of $G(x, y)$ in Hermitian polynomials gives a good description of more complicated pumping distributions. Using these functions we may approximate various forms of pumping distribution found in real samples [7]. The order in which the modes come into a state of generation as the pumping energy gradually increases is determined, on the one hand, by the different effects of pumping nonuniformities on these modes and, on the other hand, by the differences between the modes as regards diffraction losses. Since the dimensions of the generation spot were assumed to be much smaller than the transverse dimensions of the active element and the mirrors, diffraction losses were not taken into account in Eq. (1). We shall later establish the conditions under which this is valid. The integration limits in Eq. (1) may be extended to infinity, and the integrals of Eq. (1) may be reduced to tabulated functions on taking $G(x, y)$ in the form of (5) or (6). On gradually increasing $n$, the number of modes coming into a state of generation also increases. At the threshold value of $n$ for any $j, k$ mode the amplitude of the mode may be regarded as zero, while Eq. (1), taken over all the generating modes, forms a system of algebraic equations. On increasing $n$, the $j, k$ mode starts generating, and the foregoing system of equations (with its order increased by 1) now determines the generation threshold of the $j, k + 1$ mode. Further increases in the pumping level may continue until the condition

$$\sum A_{nk}^2 s_k < 1$$

used in the derivation of (1) is infringed. This method may also be used for the case of a substantial excess of pumping intensity over the threshold value by making slight (but not fundamental) changes in Eq. (1).

Figure 2 shows the calculated relationships giving the threshold values of the pumping levels for the first few modes and various degrees of nonuniformity. The threshold for the fundamental mode $TEM_{00}$ is taken as 1. We see that an increase in the degree of nonuniformity raises the thresholds of the higher modes. Thus, for a slight nonuniformity $r_x = r_y = 1.34w$ the threshold of the $TEM_{00}$ mode is 1.05; on increasing the nonuniformity to $r_x = r_y = 0.505w$ the threshold of this mode increases to 1.23. An increase in nonuniformity also changes the relationship between the amplitudes of the modes. Thus, an increase in nonuniformity by a factor of 2.7 (from $r_x = r_y = 1.34w$ to 0.475w) leads to a reduction of 3.5 times in the $A_{10}/A_{00}$ ratio. If the pumping nonuniformity is different along the x and y axes (Fig. 2c and d), the degeneracy of the modes with respect to thresholds is removed. A great nonuniformity with respect to one of the axes suppresses the modes of high order and reduces the spot dimensions along this axis. We see from Fig. 2d that for $r_x = 1.34w$ and $r_y = 0.785w$, on increasing the pumping intensity right up to $n = 1.12$, there is no change in the index of the generating mode with respect to the y axis, whereas along the x axis the indices change from 0 to 2. This leads to an increase in the radius of the generating spot along the x axis relative to that along the y axis (by a factor of 1.8). The influence of diffraction losses on the discrimination of the modes with respect to the threshold may be estimated by means of Eq. (1) on setting $G(x, y) = 1$ in this equation, i.e., regarding the pumping as uniform. By using the normalization property of the functions $u_{jk}$ it is not difficult to show that in this case Eq. (1) reduces to

$$n_k = 1 + \frac{\lambda_{nk}}{\rho} .$$

The total losses $\rho$ were taken as 0.1-0.3 [8]. The diffraction losses were determined from curves relating to passive resonators [9] for the Fresnel number corresponding to our present resonator. The results of the calculations are shown in Fig. 2a (broken lines). From a comparison between the threshold values determined by the pumping nonuniformity and the diffraction losses, we see that the former are predominant even for nonuniformities with $r_x, y = 1.34w$. A comparison between the foregoing theoretical results and