TWO-COMPONENT, CONCENTRIC, AND COPOLAR
HOMOGENEOUS SPHEROIDS IN VIRIAL EQUILIBRIUM: A
REVIEW WITH ADDITIONAL RESULTS, II

R. CAIMMI
Dipartimento di Astronomia, Universita' di Padova, Padova, Italia

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Abstract. Some results of the theory of two-component, concentric, and copolar homogeneous spheroids in virial equilibrium, derived in a previous work (Paper I) in connection with the tidal action induced by the gravitational potential of the inner subsystem on the mass distribution of the outer subsystem, have been found to hold only in the special case of similar boundaries (Caimmi and Secco, 1992). The related expressions are re-written here in a more general form, which extends their validity to subsystems of the kind considered, with all boundaries.

1. Introduction

The theory of two-component, concentric, and copolar homogeneous spheroids in virial equilibrium has been dealt with in a previous paper (Caimmi, 1991; hereafter referred to as Paper I). Further investigation (cf. Caimmi and Secco, 1992) has shown that the components of the interaction-energy tensor induced by the gravitational potential of the inner subsystem on the mass distribution of the outer subsystem coincide with the related expressions derived in Paper I only in the special case of similar boundaries. In general, a difference has been found, which yields a tensor with null trace; accordingly, no correction has to be made on the scalar interaction energy.

In summary, the results of Paper I turn out to hold concerning: (i) components of the interaction-energy tensor induced by the gravitational potential of the outer subsystem on the mass distribution of the inner subsystem; (ii) scalar interaction (and other kinds of) energies; (iii) similar boundaries (except some derivatives with respect to axis ratios). If otherwise, the related expressions have to be rewritten following Caimmi and Secco (1992) and this makes the aim of the present paper; the next sections will be numbered and named as in Paper I, and each corrected formula will be numbered as in Paper I and preceded by the index I.

2. Fixed Configurations

Let us define, in connection with the outer component,

$$\psi_{ji} = - \frac{3}{5} \frac{GM^2}{a_j} (K_{ji})_{pq},$$ (1a)

$$\Delta_{ji} = - \frac{3}{5} \frac{GM^2}{a_j} (\Delta K_{ji})_{pq},$$ (1b)
where \(G\) is the constant of gravitation; \(M\), mass; \(a\), equatorial semi-axis; \(i\), inner component; \(j\), outer component; \(p\) and \(q\), coordinate axes; \((\psi_{ji})_{pq}\) is the effective-energy tensor and \((\Delta_{ij})_{pq}\) is a tensor with null trace which makes the correction with respect to Paper I. The explicit expression of this tensor has been calculated by Caimmi and Secco (1992); accordingly, the related expressions of \((\Delta K_{ij})_{pq}\) turn out to be

\[
(\Delta K_{ij})_{pp} = \left( \frac{1}{2} \delta_{pX} + \frac{1}{2} \delta_{pY} - \delta_{pZ} \right) \frac{1}{4} \frac{1}{m} \frac{1}{y^2} \frac{1 - \eta^2}{\eta^2} \frac{\varepsilon_i (\gamma_{ij} - \varepsilon_j)}{1 - \varepsilon_j^2} \tag{2a}
\]

\[
(\Delta K_{ij})_{pq} = 0, \quad p \neq q, \tag{2b}
\]

\[
2(\Delta K_{ij})_{xx} + (\Delta K_{ij})_{zz} = 0; \tag{2c}
\]

with \(\delta_{pq}\) Kronecker symbol and \(m = M_i/M_j, y = a_j/a_i, \eta = \varepsilon_j/\varepsilon_i, \alpha_j\) and \(\gamma_j\) functions of \(\varepsilon_j\) only, as in Paper I.

The results of Caimmi and Secco (1992) also show that

\[
(W_{ji})_{pq} = (\tilde{W}_{ji})_{pq} - (\Delta_{ij})_{pq}, \tag{3a}
\]

\[
(Q_{ji})_{pq} = (\tilde{Q}_{ji})_{pq} + (\Delta_{ij})_{pq}, \tag{3b}
\]

\[
(V_{ji})_{pq} = (\tilde{V}_{ji})_{pq} - 2(\Delta_{ij})_{pq}, \tag{3c}
\]

\[
(\psi_{ji})_{pq} = (\tilde{\psi}_{ji})_{pq} - 2(\Delta_{ij})_{pq}, \tag{3d}
\]

\[
(E_j)_{pq} = (\tilde{E}_j)_{pq} - (\Delta_{ij})_{pq}, \tag{3e}
\]

where \((W_{ji})_{pq}\), \((Q_{ji})_{pq}\), \((V_{ji})_{pq}\), and \((E_j)_{pq}\), denote the interaction-energy tensor, the residual-energy tensor, the tidal-energy tensor, and the energy tensor, respectively; and quantities with the tilde superimposed relate to the special case when the boundaries of the two subsystems are similar. The results of Paper I belong to this special case instead of the general situation, as it was erroneously thought.

It is worth recalling that (Brosche et al., 1983; Caimmi et al., 1984; Caimmi and Secco, 1992):

\[
(V_{ji})_{pq} = (W_{ji})_{pq} + (Q_{ji})_{pq},
\]

\[
(\psi_{ji})_{pq} = (\Omega_{ji})_{pq} + (V_{ji})_{pq},
\]

where \((\Omega_{ji})_{pq}\) denotes the self-energy tensor; and

\[
(E_j)_{pq} = \frac{1}{2} (\psi_{ji})_{pq},
\]

provided the virial theorem in tensor form holds for subsystems.

Equations (1) and (3d) lead to

\[
(K_{ji})_{pq} = (\tilde{K}_{ji})_{pq} - 2(\Delta K_{ij})_{pq}, \tag{4}
\]