DIFFUSION APPROXIMATION IN THE THEORY OF RADIATIVE TRANSFER

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(Received 19 February, 1992)

Abstract. The transfer process of hard X-ray radiation passing through a 'cold' plasma is studied in this paper under the diffusion approximation. We call such a particular transfer process a down-Comptonization. A diffusion equation describing this process is derived and its potential applications in astrophysics and radiation physics are also pointed out.

1. Introduction

A study of the radiative transfer process in plasma is an important topic in astrophysics and radiation physics (Rubicki and Lightman, 1982). It is the exchange of energy between the radiation field and plasma that causes the variations of the emergent radiation, e.g., the spectrum, intensity, profile of an emission line, line-shift, intensity ratio of different lines, polarization, etc. For a fully-ionized plasma, the radiative transfer has a particular property, where the main way of exchanging energy between the radiation and plasma is photon-electron scattering. So far there is only a qualitative or semi-qualitative approach to the down-Comptonization (Rybicki and Lightman, 1982; Shapiro, 1972). A good approximation method was given by Kompaneets, known as diffusion approximation (Kompaneets, 1957), in which the total system of radiation field and plasma is regarded as a mixed gas consisting of the photon gas and the electron gas and the change of the frequency-spectrum of radiation field due to the electron-photon scattering is formally considered as a ‘diffusion’ process of photons in the ‘frequency-space’. Under condition $h\nu \ll kT_e \ll Me c^2$ ($h\nu$ is the average energy of a photon and $kT_e$ the average thermal energy for each electron), the dynamical diffusion equation for the photon distribution function $n(\nu) \equiv n(x)$ given by Kompaneets (1957) is

$$\frac{\partial n}{\partial t} = \frac{KT_e}{M_e c^2} N_e \sigma_T c \frac{1}{x^2} \frac{\partial}{\partial x} \left\{ x^4 \left[ \frac{\partial n}{\partial x} + n(n + 1) \right] \right\},$$

where $x \equiv h\nu/KT_e$ is the dimensionless frequency of a photon; $\sigma_T$, Thomson cross-section; $N_e$, the number density of electron gas; $n(x, t) \equiv n(\nu, t)$ is the frequency-distribution of photon gas which represents the ‘number of photons’ in each photon state.

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with frequency $v$ in unit volume, therefore, the real number density of photons in $v - v + dv$ is $n(v, t) (8\pi v^2/c^3) dv$.

However, Equation (1) cannot be used in the case when $h\nu \gg KT_e$ which is often met in X-ray and $\gamma$-ray astronomy. We call such a situation as down-Comptonization. Ross et al. (1978) suggested a new equation to replace Equation (1) for the case $KT_e \ll h\nu \ll Me c^2$,

$$\frac{\partial n}{\partial t} = \frac{KT_e}{Me c^2} N_e \sigma_T c \frac{1}{x^2} \frac{\partial}{\partial x} \left\{ x^4 \left[ n \left( 1 + \frac{KT_e}{10 Me c^2 x^2} \right) \frac{\partial n}{\partial x} \right] \right\}. \tag{2}$$

But Equation (2) is questionable because the proper form of diffusion equation must satisfy the necessary condition that $\partial n/\partial t$ should be zero if the photon gas is in thermal equilibrium. But for Equation (2), $\partial n/\partial t \neq 0$ if we insert the Planck distribution function $n(x) = (e^x - 1)^{-1}$ into Equation (2).

2. The Diffusion Equation

In this paper we present a correct form of the diffusion equation to replace Equation (2) for the case $KT_e \ll h\nu \ll Me c^2$. In the mixed gas we will assume that the electron gas has reached to thermal equilibrium due to the fact that the interaction between electrons is Coulomb's long-range force. Therefore, the Maxwellian distribution $f(p) = f_0 \exp(-p^2/2MeKTe)$ can be used to describe the electron gas. In each collision between an electron with momentum $p$ and a photon with frequency $v$, the energy and momentum conservations in the non-relativistic limit are as

$$\left( \frac{h\nu}{c} \right) n + p = \left( \frac{h\nu'}{c} \right) n' + p', \tag{3}$$

$$h\nu + \frac{p^2}{2Me} = h\nu' + \frac{p'^2}{2Me};$$

where $p'$ and $v'$ represent the momentum of electron and the frequency of photon, respectively, after the collision, $n$ and $n'$ are the directions of photon before and after the collision. The process $(p, v, n) \rightarrow (p', v', n')$ leads to a decrease of the photon number $n(v, t)$ and the inverse collision $(p', v', n') \rightarrow (p, v, n)$ leads to an increase of $n(v, t)$. The change rate of $n(v, t)$ results from Compton scattering is

$$\frac{\partial n}{\partial t} = -N_e \int d^3p \int [n(n' + 1)f(p) - n'(n + 1)f(p')] \, dW. \tag{4}$$

Equation (4) can be simplified due to the fact that the change of photon frequency in each collision is very small, $\Delta \equiv \nu' - \nu \ll \nu$. Therefore, we can expand Equation (4)