THE ABELIAN HIGGS SUNSPOT ENDOWED WITH A
LOCAL CONFORMAL SYMMETRY

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Abstract. We show that the requirement of a local conformal symmetry of the Abelian Higgs sunspot leads, at least formally, to a complex-valued electromagnetic potential, whose imaginary part is a conformal compensating potential. It is shown that there exists a fundamental difference between 'conformal' and 'ordinary' electromagnetic fields; whereas the 'ordinary' total magnetic flux of a spot is quantized its 'conformal' analogue has to vanish if the Higgs field is to be single-valued. We further stress that such a 'complex-valued' Abelian Higgs field configuration mimics quite well, under certain conditions (all the salient features of) the classical Abelian Higgs sunspot.

1. Introduction

An Abelian Higgs sunspot model (Saniga, 1990a,b, Saniga, 1992a-e; Saniga and Klačka, 1992) seems to be attractive not only for its observational support (providing a quite satisfactory and sufficiently complex description of the geometry and magnetic field structure of an isolated, fairly symmetric sunspot) but also in its theoretical aspects (being intimately related to the properties of the Sun's space-time geometry). It is especially the latter fact which turns out to be of a crucial importance in developing further this model; a massless, complex scalar field is shown to exhibit all relevant properties of the Higgs field if coupled minimally to an electromagnetic vector potential in the background of a globally conformally symmetric 'semi-metric' space-time, provided that the structure of the latter is governed by the Einstein equations with non-vanishing positive-valued cosmological constant (Saniga, 1992e). We should stress here that it is namely the requirement of a conformal symmetry which, in its global aspect, 'wakes up' the Higgs character of the above-mentioned scalar field with the help of a non-zero cosmological term (a non-linearity or self-coupling of the scalar field) and the Ricci scalar curvature of the Sun's space-time (its non-zero vacuum value; see Saniga, 1992e). We can rephrase the last statement saying that the transformation of the massless complex-valued scalar field into a Higgs field here is a 'gravitationally-induced' phenomenon.

On the other hand we have shown in detail (Saniga, 1990a,b, 1992a-c; Saniga and Klačka, 1992) that it is the Higgs field which seems to play a principal role within the framework of our sunspot theory; it not only generates a relatively strong (amounting to 3-4 kG) spot's magnetic field, but also causes both its confinement into a finite-dimensional, sharply-bounded flux tube and the total magnetic flux quantization (ensuring thereby an overall stability of the spot) to occur. It is, therefore, evident that the conformal symmetry of the Abelian Higgs

sunspots provides a specific link between gravitational and electromagnetic interactions inside sunspots. For in our most recent paper (Saniga, 1992c) we have discussed a conformal spot’s symmetry in its global aspect only the purpose of this paper is to extend our considerations having a look at what happens if we localize corresponding conformal transformations.

2. Abelian Higgs Sunspot Model; A Global Conformal Symmetry

It is instructive and, at the same time, necessary to start our reasonings reviewing characteristic features of the Abelian Higgs field configuration endowed with a global conformal symmetry. Such configuration is represented by the Lagrangian density of the form (Saniga, 1992c)

\[
\mathcal{L} = \frac{1}{16\pi\epsilon} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left( \frac{\partial \Phi}{\partial x^\mu} + i \frac{\vec{c}}{\epsilon} A_\mu \Phi \right) \left( \frac{\partial \Phi^*}{\partial x^\nu} - i \frac{\vec{c}}{\epsilon} A_\nu \Phi^* \right) g^{\mu\nu} - \frac{\hat{\lambda}}{4} \left( \Phi \Phi^* \right)^2 - \frac{2\hat{m}^2}{\hat{\lambda}} \Phi \Phi^* \]

with

\[\hat{\lambda} \equiv \frac{c^3}{2\pi G\kappa_0^4} \Lambda,\]

\[\hat{m}^2 \equiv \frac{c^3}{8\pi G\kappa_0^2} \hat{R},\]

where \( \hat{R} \) is the Ricci scalar

\[\hat{R} \equiv g^{\kappa\nu} \hat{R}_{\kappa\mu\nu} = g^{\kappa\nu} \left( \hat{\Gamma}^\mu_{\kappa\nu} - \hat{\Gamma}^\mu_{\kappa\lambda} \hat{\Gamma}^\lambda_{\nu\mu} + \hat{\Gamma}^\mu_{\sigma\nu} \hat{\Gamma}^\sigma_{\kappa\mu} - \hat{\Gamma}^\mu_{\nu\sigma} \hat{\Gamma}^\sigma_{\kappa\mu} \right),\]

and the affine connection is of a ‘semi-metric’ shape\(^1\)

\[\hat{\Gamma}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\kappa} \left( \frac{\partial g_{\kappa\mu}}{\partial x^{\nu}} + \frac{\partial g_{\kappa\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\kappa}} \right) + \left( \delta^\lambda_{\mu} \frac{\partial \ln f}{\partial x^\nu} + \delta^\lambda_{\nu} \frac{\partial \ln f}{\partial x^\mu} - g_{\mu\nu} \delta^\lambda_{\kappa} \frac{\partial \ln f}{\partial x^\kappa} \right) \equiv \Gamma^\lambda_{\mu\nu} + D^\lambda_{\mu\nu}.\]

In order to reveal that the Lagrangian density (1) possesses a global conformal invariance, i.e. that it retains its shape unchanged under the transformations (in what follows we always parametrize the Higgs field as \( \Phi(\chi^\sigma) \equiv f(\chi^\sigma) e^{i\beta(\chi^\sigma)} \))

\(^1\) The notation used in this paper is \([\ldots]\) in the sense of Misner et al. (1973). For the definition and explanation of the remaining symbols we refer the reader to Saniga (1992c,e) as well as to Saniga and Klačka (1992).