ON THE CHOICE OF OPTIMUM THICKNESS FOR A
PHOTOCATHODE IN THE CASE OF MULTIPLE
LIGHT TRANSMISSION

V. P. Vasil'ev, G. È. Kufal',
L. F. Pliev, and M. V. Fok

Recently there have been a number of papers relating to methods of increasing the sensitivity of photomultipliers by increasing the degree of absorption in the photocathode, using the principle of multiple light transmission [1-4]. The question of the optimum thickness of the photocathode, however, has hardly been treated at all. It is nevertheless quite obvious that that thickness of the photocathode which is the optimum for a single transmission may not be the optimum for the multiple transmission of light. An experimental determination of the optimum thickness requires very prolonged investigations. A more reasonable approach would be that of determining the optimum thickness theoretically, so as to find how much gain in sensitivity might be achieved by using photocathodes of optimum thickness. In order to carry out such a calculation it is essential to refine the corresponding theory of the external photoeffect so as to be able to compare this quantitatively with experiment.

The main stages in the external photoeffect may be qualitatively described as follows. Let us assume that light falls normally on to the photocathode from the direction of the glass substrate. Some of the light is reflected from the glass/photoeathode boundary, while the rest passes inside and is gradually absorbed as it approaches the photocathode/vacuum interface. On reaching this interface the light is partially reflected and the reflected component again passes through the photocathode, while a considerable proportion passes straight out.* The absorption of both the incident and the reflected wave leads to the formation of "hot" electrons, i.e., electrons having an energy exceeding that of the "bottom" of the conduction band by many kT units. If the energy is great enough, the electrons may escape from the photocathode, but for this to happen it is essential that they should reach the boundary of the latter while still having a normal component of momentum exceeding a certain specified value, determined by the photocathode material. It is well known that the surfaces of photocathodes are quite rough, with asperities considerably exceeding the de Broglie wavelength of the electrons, although much smaller than the wavelength of light. Hence to a first approximation we may neglect the wave properties of the electrons, and consider that the probability of an electron's encountering the surface of the photocathode at any particular angle will be independent of the direction of its momentum. If this momentum is "unfavorable," i.e., if the electron is unable to emerge from the photocathode despite the fact that its energy is adequate, then there is a high probability that it will emerge the next time, since after reflection from the surface of the photocathode the momentum will be directed in a more favorable manner. Simple calculations show that, if the greatest angle of incidence at which the electron is capable of emerging from the photocathode is equal to 45°, then, if there are any asperities on the surface of the photocathode having the form of tetrahedral prisms, in which the faces are inclined at an angle of 45° to the base, electrons will emerge from the photocathode for any angle of incidence. Under practical circumstances the situation is of course rather less favorable, but the asperities present on the surface of the photocathode always greatly increase the number of photoelectrons leaving it. We shall therefore consider that every photoelectron reaching the surface of the photocathode and having

*Since the thickness of the photocathode is much smaller than the wavelength of light, we can only speak of the direction of propagation of the light in the cathode very formally. However, if we use the optical constants directly determined for the photocathode rather than those relating to the bulk material, this inaccuracy may be to a large extent obviated.

sufficient energy will ultimately leave it, although, strictly speaking, the probability of this event is less than unity.

On their way from the point of origin to the surface of the photocathode, the electrons may experience various "collisions" with phonons and crystal-lattice defects. As a result of each collision the energy of a "hot" electron will diminish and the direction of its momentum will change at the same time. The mean free path of the electrons in the compounds used for making photocathodes is not yet known. It is very likely, however, that in multiple-alkali photocathodes (as in other ionic compounds) the mean free path will be several tens of angstroms, i.e., approximately an order of magnitude smaller than the thickness of the photocathode. Nevertheless, many authors [5] have derived a formula of the following type for the probability that photoelectrons generated at a depth \( x \) will emerge from the photocathode:

\[
W = A \exp\left(-\frac{x}{l_0}\right),
\]

where \( l_0 \) is the "mean depth of emergence" of the photoelectrons. This kind of formula, of course, describes the probability of the transmission of an electron through a distance \( x \) without any collisions if the mean free path is equal to \( l_0 \). However, it is as yet not quite certain to what extent this expression is applicable to the external photoeffect, and how its parameters are to be determined.

Since the physical model constituting the basis for this formula is somewhat nebulous, we shall now try to find a relationship between the quantum efficiency of the photocathode and its geometrical thickness without using Eq. (1), but simply proceeding from the physical model just described.

Let us assume that a photoelectron is only able to escape from the photocathode if it has traveled the whole distance from its point of genesis to the surface of the photocathode without any collisions, and thus without losing any energy. It is reasonable to expect that this assumption will lead us to an efficiency/ thickness relationship very similar to that derived from Eq. (1). The number of photoelectrons arising in a layer of thickness \( dx \) situated at a distance \( x \) below the surface of the photocathode (Fig. 1) equals \( kI(x)dx \), where \( k \) is the coefficient representing the absorption of light by the photocathode and \( I(x) \) is the intensity of the light at a depth \( x \). If all the directions of the momenta are equally probable, then the momentum directed at an angle to the surface of the photocathode lying between \( \phi \) and \( \phi + d\phi \) will correspond to \( 1/2 \cdot kI(x)dx \sin \phi d\phi \) electrons. The probability \( W \) that these electrons will reach the surface, i.e., that they will traverse a path \( x/\cos \phi \) without experiencing any collisions, is

\[
W = e^{-\frac{x}{l_0 \cos \phi}},
\]

while their number is equal to \( 1/2kI(x)e^{-\frac{x}{l_0 \cos \phi}} \sin \phi d\phi dx \).

After integrating this expression with respect to \( d\phi \) and \( dx \), we obtain the total number of electrons emerging from the photocathode. In order to integrate with respect to \( dx \) we must know \( I(x) \). If the light falls from the direction of the glass substrate, then

\[
I(x) = (1 - R)I_0 \left[e^{-k(L-x)} + r e^{-k(L+x)}\right],
\]

where \( I_0 \) is the intensity of the light falling on the photocathode expressed as the number of quanta; \( r \) and \( R \) are the light-reflection coefficients from the photocathode/vacuum and glass/photocathode interfaces, respectively; and \( L \) is the thickness of the photocathode. In order to obtain the value of the quantum yield \( \eta_0 \) we must divide the so-found number of electrons escaping from the photocathode by the number of incident quanta \( I_0 \). Finally we obtain the following equation for normal light incidence

\[
\eta_0 = \frac{(1 - R)e^{-kL}}{2} \left( \int_0^1 \frac{1 - e^{-\frac{L}{k\mu}}} {1 - \frac{L}{k\mu}} \frac{1 - r}{1 + \frac{L}{k\mu}} \, du + r \int_0^1 \frac{1 - e^{-\frac{L}{k\mu} - L}} {1 + \frac{L}{k\mu}} \, du \right).
\]

If the light falls obliquely, so that total internal reflection occurs at the photocathode/vacuum interface, as is required for the multiple transmission of light, then we must replace \( r \) by 1 in this equation. In addition to this, the quantity \( k \) must be replaced by \( k/\cos \psi \), where \( \psi \) is the angle of refraction of the light in the photocathode. However, owing to the high refractive index of the photocathode this angle is usually quite close to a right angle and the difference between \( \cos \phi \) and unity may be neglected.

\* This equation differs from Eq. (1) in that the term \( \cos \phi \) has been introduced. Thus the use of Eq. (1) is equivalent to the assumption that all the photoelectrons move parallel to one another along the normal to the surface of the photocathode.