Thresholds for Impact Ionisation in InSb at 77K

R.G. van Welzenis

Physics Department, Eindhoven University of Technology, NL-5600 MB Eindhoven, The Netherlands

Received 23 April 1981/Accepted 20 June 1981

Abstract. The thresholds in energy for 5 different impact ionisation processes in InSb at 77K were calculated on the basis of a critical review of the available bandstructure data for larger k values. An accurate threshold value of 243 meV ± a few meV is given for the main process. It is shown that production of light holes by impact ionisation is highly improbable. It is suggested that double ionisation and light hole initiated ionisation may be equally important in interpreting quantum efficiency data. Impact ionisation by L-band electrons may contribute significantly to the avalanche in Gunn domains, explaining the rapid quenching of the latter.

PACS: 72.20, 72.40, 79.20

When charge carriers in a semiconductor are allowed to gain sufficient kinetic energy in internal or externally applied fields or from photon absorption, they may become capable of making so-called ionising collisions. In such a process the initial mobile carrier interacts with a bound carrier in another band via their mutual Coulomb repulsion. The bound carrier is thereby excited to a higher energy state and becomes mobile at the expense of the kinetic energy of the initial carrier. Because the bound carrier leaves a hole behind in its initial state we end up with three mobile carriers. The primary carrier must have a minimum kinetic energy, called the threshold energy, for the ionisation process to be possible. This threshold can be calculated from the laws of conservation of energy and momentum.

Strictly speaking momentum conservation only holds in the absence of an electric field [1]. This is no problem in the case of InSb at 77 K, because the fields involved remain relatively low (a few times 10^4 V/m). Depending on the shape of the band involved there may exist a threshold, an anti-threshold, a saddle threshold or no threshold at all. Recently, Robbins [2] gave an extensive review on how to prove the existence and calculate thresholds theoretically. Another approach, directed towards practical calculations of threshold energies, was presented by Anderson and Crowell [3]. We will also use their calculation scheme which rests entirely on the availability of accurate and detailed band-structure data. Unfortunately the data they used for InSb were outdated and lacked the necessary precision. This resulted for instance in a threshold value for the main process in InSb (Sect. 3.1) of 0.2 ± 0.2 eV.

It is the purpose of this paper to improve this situation and by using the best available data, produce an accurate value for the threshold energy of the main ionisation process in InSb at 77 K. Moreover, we will discuss four other cases of interest.

1. Calculation Methods

Robbins [2] proved that when all the bands involved in a particular transition would be isotropic and parabolic, there exists only one single threshold energy $E_T$. This can be expressed by the simple formula

$$E_T = -\frac{m^*}{(m^*+m_2^*)}e,$$  \hspace{1cm} (1)

with $\varepsilon \equiv \varepsilon_1 + \varepsilon_2 - \varepsilon_1 - \varepsilon_2$, and $m^* \equiv m_1 - m_1 - m_2$. \hspace{1cm} (2)

The energy bands and wavevectors are described in Robbins' notation by

$$E_j = f_j(X_j) + \varepsilon_j, \hspace{1cm} k = X_j + K_j,$$
where \( \epsilon_j \) and \( \mathbf{K}_j \) are reference levels for band \( j \) (usually chosen to be at the band extremum) from which the variations in energy \( f_j \) and wavevector \( \mathbf{X}_j \) are measured. A side condition on (1) is that the wavevector reference levels cancel out, i.e.

\[
\mathbf{Q} = \mathbf{K}_1 + \mathbf{K}_2 - \mathbf{K}'_1 - \mathbf{K}'_2 - \mathbf{L} = 0.
\]

The index 1 refers to the carrier that is being ionised, 2 refers to the ionising carrier; the primes denote values before the ionisation. It should be noted that since the carriers may move from one band to another \( \mathbf{K}_i \) is not necessarily equal to \( \mathbf{K}'_i \). \( \mathbf{L} \) is a reciprocal lattice vector to allow for Umklapp processes and may be chosen such that \( \mathbf{Q} \) lies in the first Brillouin zone; \( \mathbf{q} \) is the wavevector of a phonon that may be involved.

We will also use an index \( h \) for holes. Hole energies are counted positive downward from a maximum of a valence band. The index \( T \) on the threshold value in (1) will be replaced by other symbols, pertinent to the various cases that will be considered.

Although the restrictions on the validity of (1) seem rather severe at first sight, it will turn out that the thresholds that are calculated from it are usually not very far off from the values found from less restricted calculations. This is not so much surprising if it is realised that most of the resultant carriers are near the band extrema. Furthermore, for most of the processes that are of the greatest practical importance in direct gap semiconductors both \( q = 0 \) and \( L = 0 \), because neither phonons nor Umklapp is involved. Thus \( \mathbf{Q} = 0 \) is then hardly a restriction. For these reasons we will develop (1) and use the results as an estimate for the cases that we will consider in Sect. 3.

If one has sufficiently detailed information on the bands (of any shape) to allow differentiation, specifically also further away from the band extrema, the scheme by Anderson and Crowell can be used. It can easily be proved that, irrespective of the shape of the bands, the group velocities of the resultant carriers must be equal [3]. This is a powerful basis for computer calculations. If we restrict ourselves to phononless cases and spherical bands, we have:

\[
\frac{1}{\hbar} \frac{\partial E_2}{\partial k_2} = \frac{1}{\hbar} \frac{\partial E_1}{\partial k_1} = \frac{1}{\hbar} \frac{\partial E'_1}{\partial k'_1}.
\]

This enables us to use a simplified scheme. Starting with some trial value for \( k_1 \) (or \( k'_1 \), or \( k_2 \)), presumably obtained from the previous parabolic approximation estimate, the corresponding values of \( k'_1 \) and \( k_2 \) are found from (4). The remaining \( k'_2 \) is determined by momentum conservation and (3). Once we know all the \( k \)-values the corresponding energies can be calculated and thus \( \Delta E = E_2 - (E_1 + E'_1 - \epsilon) \). This procedure is repeated for a monotonous series of \( k \) values until \( \Delta E \) vanishes or changes sign, indicating a threshold \( E_2 \) (or an anti-threshold [3]).

2. The Bandstructure of InSb at 77 K

We will discuss the central conduction band, the heavy and light hole bands and the subsidiary conduction band minimum at the L point. For these four bands we want to have tables of \( E \) and \( \partial E / \partial k \) vs. \( k \) for energies up to some 500 meV from the extremum in each band.

We will first discuss the anisotropy of the bands. It is generally accepted that the central conduction band (c) of InSb is one of the best examples of a spherically symmetric band. This is supported by experimental evidence [4], showing only a 4% anisotropy, as well as local pseudopotential calculations [5]. From the latter calculations the light hole band (v2) is seen to be fairly symmetric as well in the energy range of interest. The heavy hole band (v1) is known to have an appreciable anisotropy (Sect. 2.2), but it will turn out that this hardly influences the results of the threshold calculations. For the L band minimum we only have and need data along the A direction. So in all we feel reasonably justified in disregarding the crystalline orientation in subsequent discussions.

We will use the results of two theoretical approaches to the band structure of InSb. The well known \( \mathbf{k} \cdot \mathbf{p} \) calculations by Kane [6] can be used to any level of desired detail near \( k = 0 \). They are surprisingly good for the c and v1 bands even at larger \( k \) values. We will be using three levels of approximation. If the spin orbit splitting energy \( A > E_g \) \( kP \) where \( E_g \) is the fundamental bandgap and \( P \) the momentum matrix element between the conduction and valence bands as defined by Kane, then the solutions of the secular equation reduce to a form that is used abundantly in the literature. This is called the hyperbolic approximation.

If the assumption on the relation between \( A \) and \( E_g \) \( kP \) is not made, the secular equation reduces to a cubic equation that can be solved with standard methods. We will call this the cubic approximation. Finally, Kane applied \( \mathbf{k} \cdot \mathbf{p} \) perturbation methods to correct the results of the cubic approximation for higher bands. This is the full Kane result, which in the original paper was given for an “average” \( \langle 110 \rangle \) direction only.

The second approach are the local pseudopotential calculations by Chelikowsky et al. [5]. The aim of this type of calculations was to find the overall band shapes throughout the Brillouin zone, they thus lack fine detail. Nevertheless, it is our only source for the L-band and it turns out to yield a good description of the v2 band.

The bandgap \( E_g \) of InSb at 77 K is 225 meV [7]. The spin orbit splitting energy \( A \) is 803 meV [8]. Thus