The strength of fibrous composite plates is dependent to a significant degree on the geometry of fiber placement, i.e., on the sequence of their arrangement and orientation. To select the optimum reinforcement geometry for these composites from positions of linear fracture mechanics, it is necessary to have at one's disposal data on the local stress-strain state near reinforcing elements of the fiber-inclusion type. Consideration of the interaction between fibers, which provides for accurate computed data for the design of fibrous composites, is also required. In this connection, problems involving the elastic equilibrium of plates with inclusions that are ordered into certain systems, for example, with their periodic and double-periodic arrangement, are of special interest in the theory of the failure of composite materials.

Up till now, solutions of periodic and double-periodic problems in crack-notch theory alone have been presented most completely in the scientific literature [1-3]. An investigation of the periodic problem was recently conducted by Berezhnitskii and Stashchuk for rigid linear inclusions [4].

The present study addresses similar problems for the periodic and double-periodic systems of linear fibers of infinite stiffness, which are contained in an elastic homogeneous isotropic matrix. Analytical equations for computing the stress intensity factors near these structural stress raisers that interact with one another are presented, and their optimal orientations, which give rise to minimum stress intensity factors, determined.

Each discrete fiber of a composite plate is modeled by a thin nondeformable filament of finite length. In this case, it is assumed that the matrix in which the fibers exist is a continuous medium, which ensures an ideal bond between the fibers and the matrix material. It is also assumed that the shear modulus of the fibers exceeds that of the matrix by a factor of 100 and more; this corresponds to a composite reinforced with fibers of infinite stiffness.

To state and solve the problems considered in the study, the plane of the composite is combined with the plane of an xOy Cartesian coordinate system with respect to which the orientation and arrangement of fibers in the composite, which are ordered into a periodic or double-periodic system, are prescribed. Stress and rotations are absent in this plane at infinity; in this case, boundary conditions of the problems are given at the ends of the fibers in the composite.

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Periodic Fiber–Inclusion System in an Elastic Plane. Let us examine a periodic system of linear rigid inclusions of length 2l with centers on the real Ox axis and with equal angles of incline to it (\(\alpha_k = \alpha\), Fig. 1). The distance between neighboring defects d is constant.

Similar displacement jumps

\[
2G \frac{d}{dt} (u_n^+ + iu_n^-) = f'(t_n) \pm g'(t_n); \quad |t_n| < l
\]

are given for the contours of fibers of infinite stiffness by virtue of periodicity, while the stress jumps

\[
o_{n^=} - it_{n^=} = \rho(t_n) \pm q(t_n); \quad |t_n| < l
\]

are unknown functions. The plus and minus signs correspond to the upper and lower ends of the inclusions, t_n are points on the n-th contour of the inclusion, and G is the shear modulus of the composite's matrix material. Introducing the designations

\[
\{ -2q(t_n), -2p(t_n) \}, p^* = -1;
\]

\[
\{ 2g'(t_n), 2f'(t_n) \}, \rho^* = \kappa,
\]

where \(\kappa = 3 - 4\nu\) for plane strain, \(\kappa = (3 - \nu)/(1 + \nu)\) in the case of a generalized plane stressed state, and \(\nu\) is Poisson's ratio, we arrive at a single integral equation in terms of the unknown function \(G(\lambda, t)\)

\[
\int_{-l}^{l} [G(p, t) K(t-x, \rho^*) + \overline{G(p, t)} L(t-x)] dt = 0.5n\rho(t) F(\rho^* x);
\]

the kernels of which, according to [4, 5], assume the form

\[
K(x, \rho^*) = \frac{n\rho^*}{2d} \left( e^{i\alpha} \cotg \frac{\pi x e^{i\alpha}}{d} + e^{-i\alpha} \cotg \frac{\pi x e^{-i\alpha}}{d} \right);
\]

\[
L(x) = \frac{n}{2d} \left( e^{-i\alpha} - e^{i\alpha} \right) \left( \cotg \frac{\pi x e^{-i\alpha}}{d} - \frac{\pi x e^{-i\alpha}}{d} \cos \sec^2 \frac{\pi x e^{-i\alpha}}{d} \right),
\]

by satisfying boundary conditions (1) and (2) in a manner similar to that in [1, 2, 4, 5]. For a composite reinforced with fibers of infinite stiffness, the parameters \(\rho^* (\rho)\) correspondingly assume the value \(\kappa = -1\). If we set \(\rho^* = -1\) and \(\rho = \kappa\), we obtain an integral equation of the periodic problem for cracks.

The solution of Eq. (4) should satisfy the condition