ON THE MEASUREMENT OF THE ELECTRO-OPTICAL EFFECT IN CRYSTALS

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When an electric field is applied to a crystal, double refraction results. This leads to a phase difference \( \gamma \) between the ordinary (o) and extraordinary (e) waves. When this quantity is measured and the voltage on the crystal is known, the electro-optical coefficients, \( r_{ij} \), can be determined, since \( \gamma, E, \) and \( r_{ij} \) are related by the expression

\[
\gamma = \frac{2\pi a}{\lambda} \sigma n_0^2 r_{ij} E_j.
\]

Here \( a \) denotes the orientation of the crystal; \( \lambda \) is the wavelength of the light; the factor \( \sigma \) in Eq. (1) accounts for the crystal dimensions; \( n_0 \) denotes the refractive index; \( r_{ij} \) denotes the tensor of the electro-optical effect; and \( E_j \) is the electric field strength.

The easiest method of measuring the electro-optical effect utilizes a \( \lambda/4 \) plate as described in [1]. The measurement of the electro-optical effect is simplified if instead of the phase difference \( \gamma \) the voltage \( U_{\lambda/2} \) necessary to create a phase difference of \( \lambda/2 \) is measured and the electro-optical coefficients are subsequently calculated. Figure 1 shows the block diagram of a setup for measuring \( U_{\lambda/2} \). The light ray from a SPM-1 monochromator passes through an optical system consisting of a polarizer, a chromatic \( \lambda/4 \) plate, the sample to be measured, an analyzer, and a photomultiplier (7) for signal detection. After the photomultiplier, the constant and the variable components of the output voltage enter a multiplier unit, into which the signal indicating the voltage on the crystal is also fed. The multiplier unit has an output with an indicating meter and its deflection is proportional to \( U_{\lambda/2} \). The analyzer transforms the change in the light polarization into intensity variations of a linearly polarized light ray according to

\[
I = I_0 \sin^2 \gamma/2.
\]

A change in light intensity at the multiplier input leads to a voltage at the terminating load of the form

\[
v = v_0 \sin^2 (\gamma_0 + \gamma_m \sin \omega t/2),
\]

where \( \gamma_0 \) denotes the initial phase shift defined by the optical system; and \( \gamma_m \) is the maximum phase shift due to the electric field. Equation (3) is valid if the gain of the photomultiplier is independent of the frequency with which the intensity of the incident light varies. It is convenient to rewrite Eq. (3) in the following form:

\[
\eta = \frac{v}{v_0} = \sin^2 (\gamma_0 + \gamma_m \sin \omega t/2)
\]

and to expand this expression in a series in terms of integer Bessel functions:

\[
\gamma = 1/2 \left[ 1 - \cos \gamma_0 \left( I_{\lambda} (\gamma_m) + \sum_{n=1}^{\infty} J_{2n} \sin 2n \omega t \right) + \sin \gamma_0 \sum_{n=0}^{\infty} J_{2n+1} (\gamma_m) \sin (2n+1) \omega t \right].
\]

We note that for \( \gamma_0 = (2n + 1)\pi/2 \), the expansion contains only odd harmonics of the modulating frequency \( \omega \)

\[
\gamma = 1/2 \left[ 1 + \sum_{n=0}^{\infty} J_{2n+1} \sin (2n + 1) \omega t \right];
\]

for \( \gamma_0 = n\pi \) we have only even \( \omega \) values:

\[
\gamma = 1/2 \left[ 1 - J_0 (\gamma_m) - \sum_{n=1}^{\infty} J_{2n} \sin 2n \omega t \right].
\]
The general case of Eq. (5) ($\gamma_0$ assumes arbitrary values except those indicated above) can be considered as a combination of Eqs. (6) and (7). These expressions simplify considerably if $\gamma_m \ll 1$. Then, if we represent $\gamma_m$ by the voltage on the crystal, $U_m$, and by $U_{\lambda/2}$, we have $\gamma_m = \pi U_m / U_{\lambda/2}$, which is obtained from Eq. (1). The increment $\Delta \eta = \eta - \eta_0$ in Eqs. (6) and (7) (where $\eta_0$ denotes the values at the points $\gamma_0 = (2n + 1)\pi/2$, $\gamma_0 = n\pi$) can be expressed by the voltage change at the crystal:

$$\Delta \eta = \pi/2 (U_m / U_{\lambda/2}), \quad \gamma_0 = (2n + 1)\pi/2,$$

$$\Delta \eta = (\pi U_m / 2U_{\lambda/2})^2, \quad \gamma_0 = n\pi. \tag{8} \tag{9}$$

At the same time, we can express $\Delta \eta$ in the following form (see Fig. 2): $\Delta \eta = \Delta v / v_m$, where $\Delta v$ denotes the voltage increment at the photomultiplier input due to the double refraction generated by the field. When we replace $\Delta U_m$ by $U_m$ and $\Delta v$ by $v_m$, we obtain from Eqs. (8) and (9)

$$U_{\lambda/2} = \pi/2 \frac{v_0}{v_m} U_m, \quad \gamma_0 = (2n + 1)\pi/2, \tag{10}$$

$$U_{\lambda/2} = \pi/2 \sqrt{v_0/v_m} U_m, \quad \gamma_0 = n\pi. \tag{11}$$

Fig. 2. Characteristics of the light transmission $\eta$ and sensitivity $\kappa$ of the optical system: $U_{\lambda/2}$ amplitude of voltage on crystal; $v_m$ voltage change at photomultiplier load caused by $U_m$; $v_{0/2}$ constant voltage determined by the $\lambda/4$ plate.