Detuning effects of FM mode-locking in atmospheric CO$_2$ lasers

W. J. Witteman, A. H. M. Olbertz
Department of Applied Physics, Twente University of Technology, Enscherle, The Netherlands

Received 22 October 1979; revised 2 January 1980

We describe a study of the detuning of the intracavity FM modulation of the homogeneously broadened laser applied to CO$_2$ systems. The analysis is done by working in the frequency domain and by describing the field in terms of discrete amplitudes and phases. The coupled mode equations are solved for modulation frequencies near the axial mode-separation frequency. The asymmetry of the mode-locking behaviour due to the dispersive behaviour of the active medium is verified experimentally. We have measured both the pulse and the phase shift of the pulse with respect to the modulation signal as a function of detuning. The experimental results show clearly the asymmetry with detuning, in agreement with theory.

1. Introduction

The problem of mode-locking and pulse forming by means of intracavity AM and FM modulation has been treated in the past by several authors [1-8]. For homogeneously broadened systems one may say that the problem has been approached from two different sides. One way is to work completely in the time domain and to assume a Gaussian optical pulse inside the resonator [4, 5]. The pulse parameters are then obtained from the requirement that after a complete round-trip through the resonator the pulse is unchanged. The other approach is to work in the frequency domain and to describe the field in terms of discrete mode amplitudes and phases [1-3].

In the present paper we analyse the detuning effects of FM mode-locked CO$_2$ lasers by working in the frequency domain. We derive as a function of the detuned modulation frequency spectrum, pulse phase shift with respect to the modulation, the gain, and the shift of the centre modulation frequency of the pulse. The theoretical results are analogous to those obtained in the time domain approach [4]. The difference is that we start from the discrete set of oscillating modes and show that the amplitude and phase distribution of these modes satisfy a Gaussian form. Experimentally we measured for the case of a CO$_2$ TEA laser the dependence of the pulselwidth on the frequency detuning for two different modulation signals. The dependence of the phase shift of the pulse with respect to the periodic modulation on the detuning is measured for 1 and 2 atmosphere laser mixtures. The experimental results show clearly a strong asymmetry in accordance with the analysis. Small detuning effects turn out to have a large bearing on the pulse parameters.

2. Derivation of the basic equation

In the following we shall work in the frequency domain, in order to obtain an analytical solution for a stationary phase-locked, internally FM modulated, laser system.

A fruitful analysis is to start with the self-consistency equations of Lamb [10], which describe the effect of an arbitrary polarization upon the electric fields. In the case of mode-locking the polarization includes contributions of both the active medium and the modulation. We shall follow the notation of Harris and McDuff [1] in order to arrive at an equation for the complex field amplitudes using the axial modes as a basic set.
The modulation is an electro-optic device that produces a periodic phase modulation. For our system we used a CdTe crystal of 40 mm length and an oscillator with a maximum output voltage of 2000 V. The phase modulation of such a modulator can be described by $\Delta \chi(z, t)$, the in-phase component of the susceptibility:

$$\Delta \chi(z, t) = \Delta \chi'(z) \cos \omega_m t$$

(1)

where $\omega_m$ is the modulation frequency.

In the case where the modulator is located near the out-coupling mirror at $z = d$ and its length is much smaller than the distance $d$ between the mirrors we finally find the basic equation for the electric field:

$$E_{n+1} e^{i\phi_{n+1}} + E_{n-1} e^{i\phi_{n-1}} = \frac{1}{\delta} E_n e^{i\phi_n} \left[ -\left( \frac{2d}{c} \frac{\phi_n}{\omega} - \frac{2d}{c} n\Delta \omega + \psi_n \right) - iG_n + iL \right]$$

(2)

where $\phi_n$ is the phase of the $n$th mode, $L$ is the single pass power loss, and

$$\psi_n = \frac{\omega_n \chi_n d}{c}$$

(3)

is the phase retardation of the $n$th mode caused by the active medium

$$G_n = -\frac{\omega_n \chi_n'' d}{c}$$

(4)

being the single pass power gain of the $n$th mode through the medium;

$$n\Delta \omega = \Omega_n - \omega_n$$

(5)

being the difference between the $n$th empty cavity mode and the real mode; and

$$\delta = \frac{\omega_n}{c} \int_0^a \Delta \chi' \sin k_n z \sin k_n \pm 1 z \, dz$$

(6)

being the modulation parameter.

In deriving Equation 2 we have omitted the integrals of the form

$$\int_0^a \Delta \chi'(z) \sin k_n z \cos k_n \pm 1 z \, dz$$

because they are negligible for the position of the modulator near $z = d$.

3. Steady-state solution for the homogeneously broadened line

In the following we look for an analytical solution of Equation 2 for the case in which we are dealing with a homogeneously broadened line. Due to the interactions of the radiation field with the inverted medium the gain profile changes over its full frequency range. In general it turns out that the frequency width of the pulse is small as compared to the line-width. This is illustrated in Fig. 1, where the gain and the mode amplitudes are plotted against the frequency. This means that in the frequency region of interest we may, to a good approximation, expand the gain profile and the phase retardation of such a line profile to the second order.

This can be done in terms of the mode number $n$. Thus the gain and the phase retardation for the $n$th mode will be given by

$$G_n = G_0 + nG_1 + \frac{1}{2} n^2 G_2$$

(7)

$$\psi_n = \psi_0 + n\psi_1 + \frac{1}{2} n^2 \psi_2$$

(8)

In looking for an analytical solution of Equation 2 it has been argued [11] that for small detuning the quadratic terms in Equation 2 take over the effects of the linear terms on the field distribution and the