Optical information processing by the Fabry-Perot resonator

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Received 10 November 1975

Properties of the Fabry-Perot resonator as a part of an optical imaging system are discussed from the point of view of optical information theory. Information capacity of the system is derived and expressed in terms of the number of degrees of freedom of the optical wave field at the output. It is pointed out that the Fabry-Perot resonator is well suited for processing of image signals having a periodic structure.

1. Introduction
As it has been shown in recent publications [1, 2] the Fabry-Perot resonator can be potentially useful in optical data processing applications. Therefore it may be worthwhile to analyse the transmission properties of this resonator, if it acts as a spatial filter. We take into consideration its ability to transfer image information and its resolving power in spatial domain. The theory is formulated in terms of the number of degrees of freedom of the wave field at the output of the resonator and is based on Abbe’s theorem as well. In accordance with the circular symmetry of the optical transfer function of the resonator it is advantageous to use the sampling theorem in polar coordinates [3].

The principle formulae which are valid for the Fabry-Perot resonator are presented in Section 2. The theory of transmission of an image signal by the resonator is developed in Section 3. A simple expression for the limit of resolution of the resonator is derived and its information capacity in terms of the number of degrees of freedom of the wave field is determined. Some current possible applications are listed and discussed in Section 4.

2. The Fabry-Perot resonator
The transmissivity of the Fabry-Perot resonator (FPR) is given by the formula first derived by Airy.

$$T_{FP}(\lambda, \gamma) = \frac{T^2}{1 + R^2 - 2R \cos \Delta \phi}; \quad \Delta \phi = \frac{4\pi L}{\lambda} \cos \gamma.$$ (1)

We denote by $R$ and $T$ the reflectivity and transmissivity of the mirrors, respectively, $(R + T = 1)$, and the distance between them by $L$; $\lambda$ is the wavelength and $\gamma$ is the angle of incidence of light. Transmission is maximum when $\Delta \phi = 2m\pi$, where $n$ takes on integral values and represents the order of interference. For fixed values of $L$, $R$ and $\lambda$, the transmissivity $T_{FP}(\gamma)$ can be varied by varying $\gamma$. If the resonator is illuminated with monochromatic light from an extended source and the transmitted light is observed in the focal plane of a lens, a set of concentric rings is produced corresponding to the values of $\gamma$ that satisfy the condition

$$\gamma_m = \sqrt{\left(\frac{m\lambda}{L}\right)}; \quad m = 0, 1, 2 \ldots$$ (2)

where $m$ is the ring number counting from the centre. It follows from the Airy’s function that the sharpness of these diffraction maxima is finite and can be measured by their half-width. From the condition $T_{FP}(\gamma) = 1/2$ for $m = M$ we obtain from Equation 1
This expression is valid for \( \gamma_M \leq 0.30 \). From Equation 3 we can see that when \( M \) increases then \( \delta \gamma_M \) decreases. We shall use these formulae in the next parts of the paper.

### 3. Transmission of image information by the Fabry-Perot resonator

As it is well known, the ability of an optical system to transfer image information can be evaluated in terms of degrees of freedom of the wave field at the output of the system [4]. This approach can be also adapted for an optical system, where the FPR acts as a filter of spatial frequencies of input signals.

Let us consider a band-limited imaging system which is linear and spatially invariant. An input optical signal (object function) \( f(r, \phi) \) is equal to zero for all \( r > r_0; r, \phi \) denote the polar coordinates in the object plane. For the Fourier transform of an image function \( u(r, \phi) \) holds

\[
\mathcal{F}[u(r, \phi)] = F(p, \theta).G(p, \theta)
\]

where \( \mathcal{F}[f(r, \phi)] = F(p, \theta) \) and \( G(p, \theta) \) is the optical transfer function (OTF) of the optical system under consideration; \( p, \theta \) denote the polar coordinates in the plane of spatial frequencies. In [3] there has been calculated the number of degrees of freedom, connected with the function \( u(r, \phi) \) for two specific shapes of the OTF

(a) \( G(p, \theta) = G(p) = \text{circ}(\rho, \rho_0) = \begin{cases} 1 & \text{for all } \rho \leq \rho_0 \\ 0 & \text{for } \rho > \rho_0 \end{cases} \)

(b) \( G(p, \theta) = G(p) = \text{ring}(\rho, \rho_m, \Delta \rho_m) = \begin{cases} 1 & \text{for } \rho_m - \frac{\Delta \rho_m}{2} \leq \rho \leq \rho_m + \frac{\Delta \rho_m}{2} \\ 0 & \text{for all other } \rho. \end{cases} \)

The corresponding numbers of degrees of freedom are as follows

(a) \( N_i = (\rho_0 r_0/2 + 1)^2 \)

(b) \( N_i = 2r_0 \rho_m \left( r_0 \frac{\Delta \rho_m}{4} + 1 \right) \)

Equations 7 and 8 can be used, together with Equations 2 and 3 as the basis for our calculations.

The band-limited imaging system (linear and spatially invariant), where the FPR acts as a filter of spatial frequencies of the image signal, behaves like an optical system whose properties can be described by the following OTF

\[
G(p) = \text{circ}(\rho, \rho_0) + \sum_{m=1}^{M} \text{ring}(\rho, \rho_m, \Delta \rho_m)
\]

where

\[
\rho_0 = 0.8\pi \sqrt{\frac{1 - R}{\lambda L}}
\]

\[
\rho_m = 2\pi \sqrt{\frac{m}{\lambda L}}
\]

\[
\Delta \rho_m = \frac{2\pi}{1 - \frac{m \lambda}{2L}} \left[ \sqrt{\frac{(1 - R)(1 - m \lambda/2L)}{\pi \sqrt{R}}} + 2m - \sqrt{2m} \right].
\]

The value of \( M \) depends on the aperture of the optical system. Equation 9 is only approximate because of the sharp cut off of the diffraction maxima, but for \( R \geq 0.5 \) this approximation is quite satisfactory.