Reflection of a pulse by multiple-dielectric layers

T. SUZUKI
Department of Electrical and Communication Engineering, Faculty of Engineering, Osaka University, Suita, Osaka, Japan 565

E. OGAWA
Yokosuka Electric Communication Laboratory, N.T.T., Yokosuka, Kanagawa, Japan 238-03

H. FUJIOKA
Electron Beam Laboratory, Faculty of Engineering, Osaka University, Suita, Osaka, Japan 565

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The reflection of a pulse by multiple-dielectric layers is treated theoretically. The reflected wave is obtained by expanding the reflection coefficient of an elementary plane wave in a series, including the special case for which total reflection occurs. The pulsed waves reflected by two-dielectric slabs are considered in detail both analytically and numerically. The effect of a carrier frequency of a pulse-modulated carrier wave on the reflected wave form is also discussed.

1. Introduction

The reflection and transmission of electromagnetic waves by multiple-dielectric layers has been considered so far as one of the fundamental problems of electromagnetic wave theory [1]. This problem has also been investigated from practical point of view in connection with, for instance, a dielectric slab waveguide and wave launching into thin-film optical integrated circuits.

Several investigators have studied pulse reflection by a semi-infinite plasma medium and a plasma slab [2-4], and L. M. Brekhovskikh has treated pulse reflection by a semi-infinite medium and the propagation of a sound pulse in a liquid layer [5]. The reflection of a pulse by multiple-dielectric layers, however, has not been treated yet.

As an example of pulse reflection by multiple-dielectric layers, the present paper analyses pulse reflection by two dielectric slabs by expanding the reflection coefficient of an elementary plane wave in a series, including the special case where total reflection occurs at any boundary surface in multiple-dielectric layers. Using this method, the mechanism of pulse reflection by multiple-dielectric layers is physically well explained.

The reflection of a pulse-modulated carrier wave by two dielectric slabs is also investigated with a detailed discussion about the effect of a carrier frequency on the reflected wave form.

2. Incident and reflected pulsed waves

Fig. 1 shows the system to be considered in which the regions 1 and $N$ are the semi-infinite dielectric media whose permittivities are $\varepsilon_1$ and $\varepsilon_N$ respectively, whereas regions 2, 3, $\ldots$, $N-1$ are the dielectric layers of thickness $d_n$ and permittivity $\varepsilon_n$, where $n = 2, 3, \ldots, N - 1$. The system is assumed to be uniform in the $x$ and $y$ directions. Suppose that the plane scalar pulsed wave is coming through region 1 onto the multiple-dielectric layers at an incident angle $\theta$ with phase velocity $c = 1/\sqrt{(\varepsilon_1\mu_0)}$.© 1975 Chapman and Hall Ltd.
The incident pulse shape is assumed to be either Bell or Gaussian, expressed as

\[ F_i(\xi_i) = \frac{e}{\xi_i^2 + \zeta_i^2} \quad \text{(Bell type)}, \]

\[ F_i(\xi_i) = \frac{1}{\sqrt{\pi} w} \exp\left(-\frac{\xi_i^2}{w^2}\right) \quad \text{(Gaussian type)} \]

where

\[ \zeta_i = \frac{x \sin \theta - z \cos \theta}{c} - t . \]

The constants \( e \) and \( w \) are the parameters characterizing the shape of an incident pulse. The wave form in Fig. 1 shows the incident pulse form at the time \( t = -l/c \). The wave surface of the maximum amplitude \( (\zeta_i = 0) \) will arrive at the origin of the co-ordinate system at \( t = 0 \).

By means of Fourier integration, the incident pulse can be expressed as a sum of the elementary plane waves as follows:

\[ F_i(\xi_i) = \frac{1}{2} \int_{-\infty}^{\infty} \Phi(\omega) \exp(-j\omega \xi_i) \, d\omega = \text{Re} \int_{0}^{\infty} \Phi(\omega) \exp(-j\omega \xi_i) \, d\omega , \]

where

\[ \Phi(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} F_i(\xi_i) \exp(j\omega \xi_i) \, d\xi_i \]

is the frequency spectrum of the incident pulse, given by

\[ \Phi(\omega) = \exp(-e|\omega|) \quad \text{(Bell type)} \]

\[ \Phi(\omega) = \frac{1}{\pi} \exp\left[-(w\omega/2)^2\right] \quad \text{(Gaussian type)} . \]