A VARYING COSMOLOGICAL TERM AS A LINK BETWEEN COSMOLOGY AND MICROPHYSICS

(Letter to the Editor)

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Abstract. The Weinberg relation (which connects the Hubble constant $H$ to the mass of a typical elementary particle) is an empirical relation hitherto unexplained. I suggest an explanation based on the Zel'dovich energy tensor of vacuum in a Robertson-Walker universe with constant deceleration parameter, $q = \text{const}$. This model leads to
(a) the Weinberg relation,
(b) a varying cosmological term $\Lambda$ scaling as $H^2$,
(c) a varying gravitational constant $G$ scaling as $H$,
(d) a matter creation process throughout the universe at the rate $10^{-47} \text{ g s}^{-1} \text{ cm}^3$,
(e) a deceleration parameter in the range $-1$ to $\frac{1}{2}$, which allows a horizon-free universe and makes the law $G/H = \text{constant}$, consistent with the Viking lander data on the orbit of planet Mars.

In a spatially flat, matter-dominated universe with Robertson-Walker (RW) space-time metric the gravitational equations of the general relativity theory with $\Lambda$ term lead to (Narlikar, 1983)

\[ 3 \left( \frac{\dot{S}}{S} \right)^2 = 8\pi GU + \Lambda \]  

(1)

\[ 2 \frac{\ddot{S}}{S} + \left( \frac{\dot{S}}{S} \right)^2 = \Lambda \]  

(2)

where $\dot{S} = \frac{dS}{dt}$, $t$ is the cosmological time, $S$ is the expansion factor, $U$ is the mean mass density, $G$ is the gravitational coupling. From Equations (1) and (2) one gets

\[ \left( \frac{\dot{S}}{S} \right)^2 - \frac{\ddot{S}}{S} = 4\pi GU . \]  

(3)

Let us now assume that $S = S(t)$ is given by a power law

\[ S \propto t^a , \]  

(4)
which gives
\[
\frac{\dot{\mathcal{S}}}{\mathcal{S}} = \frac{a}{t},
\]
\[\tag{5}\]
\[
\frac{\dot{\mathcal{S}}}{\mathcal{S}} = \frac{a(a - 1)}{t^2}.
\]
\[\tag{6}\]
Using Equations (5) and (6), Equations (3) and (2) give, respectively
\[
4\pi G U t^2 = a,
\]
\[\tag{7}\]
\[
\land t^2 = a(3a - 2) = (1 - 2q)(1 + q)^{-2}
\]
\[\tag{8}\]
where \(q = -S\ddot{S}(\dot{S})^{-2}\) is the deceleration parameter. From the energy tensor of the quantum field theory polarized vacuum Zel’dovich finds (Zel’dovich, 1968)
\[
\land = (G m^3)^2 \left(\frac{c}{h^2}\right)^2
\]
\[\tag{9}\]
where \(c\) is the speed of light, \(h\) is the Planck constant, and \(m\) is the mass of a typical elementary particle. From this and from Equations (8) and (5) one gets
\[
m = \left(\frac{H h^2}{c G}\right)^{1/3} \left(3 - \frac{2}{tH}\right)^{1/6},
\]
\[\tag{10}\]
where \(H = \dot{S}/S\) is the Hubble parameter. As \(tH \sim 1\), we have
\[
m \sim \left(\frac{H h^2}{c G}\right)^{1/3}.
\]
\[\tag{11}\]
This is a well-known empirical relation (Weinberg, 1972) that now has a new possible explanation: it is a simple and straightforward consequence of the Zel’dovich quantum approach to \(\land\), in an isotropic flat space expanding with the simple law (4).

The mass-energy conservation law must be dropped. The relativistic quantum field theory, cornerstone of Equation (9), requires \(c\) and \(h\) to be time-independent; then Equations (8) and (9) give
\[
G m^3 \propto t^{-1}.
\]
\[\tag{12}\]
If \(\dot{m} \neq 0\) the mass conservation law is violated at the atomic level; if, however, \(\dot{m} = 0\) then a particle creation process occurs on cosmological scale because Equation (12) gives