NON-VACUUM COSMOLOGICAL SOLUTIONS OF 5D VARIABLE
MASS THEORY OF GRAVITY

J.C. CARVALHO, J.A.S. LIIMA and O.R. NELSON
Departamento de Física, UFRN,
Natal, RN, Brazil

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Abstract. A class of non-vacuum expanding cosmological solutions of Wesson's 5D theory of
gravity with variable rest mass is derived. The models are spatially homogeneous and isotropic and
the source of gravitation is a pressureless fluid (dust) plus a cosmological constant term. The general
and unified solution is found for the equations and some properties of different limiting cases are
studied. Particularly, it is shown that for null cosmological constant the predicted age of the universe
is smaller than the ones of the 4D FRW models.

1. Introduction

The invariance of the laws of physics under changes of coordinates is a well
established requisite of any theory. However, when one thinks of invariance under
changes of scale this is not so. Many different theories of gravity have been
proposed in which either the gravitational constant $G$ and/or the rest masses of the
objects vary with time. These theories, which are alternatives to Einstein's general
theory of relativity, have the desirable property of scale invariance, besides of being
covariant as well.

Some years ago, Wesson (1983, 1984) discussed the difficulties encountered
by these different approaches and proposed a variable mass theory of gravity. The
mass is regarded as a geometrical coordinate in a continuum 5D space-time-
mass manifold so that the usual 4D Einstein's theory is embedded in it. The fifth
coordinate is closely related to the mass $m$ through $x^4 = Gm/c^2$, where the
gravitational constant $G$ and the velocity of light $c$ are true constants.

As it was pointed out by McCrea (1978), in a theory with time dependent
gravitation it seems more plausible to let the mass change with time rather than the
gravitational constant $G$, since $m$ is the source of the field. Wesson's theory fulfill
this requirement and it is also claimed to be invariant under scale changes. Gron
and Soleng (1988) have contested the latter but they agree that in fact the theory is
free of scale and that the idea of having the variation of gravitation due to higher
dimensions might be correct. Other aspects of the theory were discussed by Wesson
models with variable rest mass, in the spirit of Wesson's theory, have been proposed
by Carvalho and Lima (1991) while the existence of classical analogs of the 5D
theory has also been investigated (Carvalho and Lima, 1992; Waga, 1992).

As it happens in many high dimensional theory of gravity, the field equations
of Wesson's theory are taken to be the 5D analogs of the 4D Einstein's equations,
namely $G^{ij} = \chi T^{ij}$, where $\chi$ is the 5D Einstein constant and $i$ and $j$ run from 0 to 4. In the framework of the Wesson’s theory, $\chi$ has been erroneously fixed to the same value of the General Relativity in four dimensions (Wesson, 1984; Gron, 1988; Ma, 1990). As it is well known, in $n + 1$ dimensions we have $\chi = (n - 1)\Omega_n G_n$, where $\Omega_n = 2\pi^{n/2}/\Gamma(n/2)$ and $G_n = G \times (\text{length})^{n-3}$ are the $n$-dimensional solid angle and gravitational constant, respectively. For Wesson’s theory the above expression gives $\chi = 6\pi^2 G_4$ and in the case of Einstein’s theory $\chi = 8\pi G$ as expected.

Some cosmological solutions of the theory have been found by Wesson (1986, 1988), Chatterjee (1987) and Fukui (1987, 1988) using the vacuum field equations $G^{ij} = 0$. Gron (1988) obtained a complete cosmological scenario by deriving vacuum, radiation and matter dominated universes. In this work we study exact cosmological solutions for dust models plus a $\Lambda$-term. We focus our attention on the 5D equations in the special case when the metric coefficients are functions of time but not of the mass coordinate. This is equivalent to suppose space-mass homogeneity. The general solutions are obtained and an analysis is made of their different limits.

2. The Solutions of 5D Equations in Presence of Matter and $\Lambda$-Term

We start by writing the line element for a homogeneous continuum “space–time–mass” in the form

$$\text{ds}^2 = c^2 \text{d}t^2 - R^2 (\text{d}x^2 + \text{d}y^2 + \text{d}z^2) - W^2 \text{d}\phi^2. \quad (1)$$

Here, $\phi = Gm/c^2$ and $R$ and $W$ are functions of the time $t$ and the mass $m$. Hereafter we make the velocity of light equals to one. For the line element (1), the non-zero components of 5D Einstein’s tensor with cosmological constant $\Lambda$ are:

$$G_{00} = (-3WR'R'' + 3WR'R' - 3WR'R^2 - 3WR'W' - 3WR'R^2 - 3WR'W' - 3WR'R^2 - 3W^3 R^2 - 3W^3 R^2 - 3W^3 R^2 - 3W^3 R^2 - 3W^3 R^2) (R^2 W^3)^{-1}, \quad (2a)$$

$$G_{11} = (2WR'R'' + WR'R' - 2WR'W' + 2WR'R'R - 2WR'R'R - 2WR'R'R - 2WR'R'R - 2WR'R'R - 2WR'R'R - 2WR'R'R) (R^2 W^3)^{-1}, \quad (2b)$$

$$G_{04} = (3WR'R' - 3WR'R') (R W)^{-1}, \quad (2c)$$

$$G_{44} = (-3R^2 - 3WR'R'R - 3WR'R'R - 3WR'R'R - 3WR'R'R - 3WR'R'R - 3WR'R'R) R^2. \quad (2d)$$

The prime and the dot denote differentiation with respect to mass and time, respectively.

The energy–momentum tensor $T^{ij}$ is constructed as in Einstein’s theory but making $i$ and $j$ running from 0 to 4 (Wesson, 1983). For a perfect fluid with negligible pressure (dust) one has,

$$T^{ij} = \rho u^i u^j, \quad (3)$$