Nuclear Magnetic Susceptibility of Solid $^3$He with $^4$He Impurities

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The nuclear magnetic susceptibility of bcc solid $^3$He with a small concentration of $^4$He is calculated in the high-temperature approximation. The exchange Hamiltonian used for the system is that obtained from a Hubbard-like cell model developed in earlier work. A brief discussion is also given of our general approach to the study of the magnetic properties of impure solid $^3$He as compared to other approaches to the problem.

1. INTRODUCTION

It is well known that the large zero-point motion of atoms in solid helium plays an important role in determining the properties of the solid. This large zero-point motion arises because the mass of a helium atom is small and the attractive part of the interatomic potential is weak. In solid $^3$He the zero-point motion is what causes a spin ordering (antiferromagnetic). It is further found experimentally$^{1,2}$ that several magnetic properties of solid $^3$He are affected by (spinless) $^4$He atoms present as impurities. With these facts in mind we have developed in earlier work$^3$ a Hubbard-like cell model for impure solid $^3$He, starting with the many-body Hamiltonian for the system; the model Hamiltonian describes the hopping of atoms in the lattice. A canonical transformation method was used to bring the Hamiltonian to a "blockwise" diagonal form. As a special case, viz., in the absence of vacancies and doubly occupied sites in the lattice, it was possible to derive explicitly an effective exchange Hamiltonian (similar to the Hamiltonian written down recently by Guyer and Zane$^6$). This last Hamiltonian was used, in the framework of a modified three-bath model, to calculate $R$ (the ratio of the exchange and Zeeman specific heats $C_{ex}$ and $C_z$), $T_2$ (the transverse relaxation time), and $\tau_{z,ex}$ (the Zeeman–exchange relaxation time) as functions of the $^4$He impurity concentration $x$, for small $x$. Agreement with experiment was found to be reasonable.

In Sections 2 and 3 we calculate the nuclear magnetic susceptibility $\chi$ from the effective exchange Hamiltonian using the so-called high-temperature approximation. In Section 2 we find $\chi$ only up to $O(T^{-2})$, and express it in the Curie–Weiss form $\chi = C/(T + \theta)$, in order to see how the parameters $C$ and $\theta$ depend on $x$. 

81
We comment on the results found and on the experimental situation. In Section 3 we carry out the calculation of $\chi$ up to $O(T^{-4})$, for theoretical reasons given at the beginning of that section. Section 4 is a general discussion on the use of the present approach for finding the $x$ dependence of various magnetic properties of impure solid $^3$He, as compared to other approaches.

2. CURIE–WEISS FORM FOR $\chi$

The effective exchange Hamiltonian we are concerned with may be written in the form

$$H_0 = \frac{J}{2} \sum_{i,j} (\mathbf{I}_i \cdot \mathbf{I}_j) + \frac{J''}{2} \sum_{i,j,\sigma} D_{i\sigma} D_{j\sigma}$$

where $\mathbf{I}_i$ is the usual spin operator at the lattice point labelled by $i$, and $D_{i\sigma} = c_{i\sigma} a_i = (D_{i\sigma})^*$; $c_{i\sigma}$ and $a_i^+$ are fermion ($^3$He) and boson ($^4$He) creation operators, respectively. The summations in (1) run over all nearest-neighbor pairs $(i, j)$. The exchange constants $J$ and $J''$ are given by $4T^2/\varphi$ and $-4TT'/\varphi'$ respectively: $T[T']$ is the matrix element describing the hopping of a $^3$He atom [$^4$He atom] between two nearest-neighbor sites; $\varphi[\varphi']$ is the hard-core repulsion energy between two $^3$He atoms [$a$ $^3$He atom and a $^4$He atom] in the same cell. ($J > 0$ as is appropriate for antiferromagnetic solid $^3$He.) We shall take $N$ to be the total number of atoms in the lattice, $v$ of these being $^4$He atoms; $x = v/N \ll 1$. We denote by $z$ the number of nearest neighbors per atom.

Adding the Zeeman energy in the presence of an external magnetic field $B$ in the $z$ direction, we write the Hamiltonian for the problem as

$$H = H_0 + H_z$$

where

$$H_z = \gamma B \sum_i I_i^z$$

$\gamma$ being the gyromagnetic ratio of a $^3$He nucleus. The susceptibility $\chi$ is calculated following a standard procedure. We have

$$\chi = \frac{kT}{N} \left( \frac{\partial^2 \ln Z}{\partial B^2} \right)_{B=0}$$

where the partition function $Z$ is given by

$$\text{Tr}\{\exp(-H/kT)\} = \langle \exp(-H_0/kT) \rangle_D$$

with $D = \text{Tr}\{\exp(-H_z/kT)\}$. (We have used the fact that $[H_0, H_z] = 0$). The bar over any operator $A$ denotes the average

$$\text{Tr}\{A \exp(-H_z/kT)\}/D$$