A Model Study of the Pressure Variation of the Superconducting Transition Temperature

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The volume dependence of the Morel–Anderson prescription for the superconducting transition temperature is calculated. The resulting expressions depend only upon atomic constants, the specific-heat effective mass, and the Grüneisen constant. No undetermined parameters enter. On the basis of the model, the full range of observed pressure dependences of $T_c$ can be qualitatively understood. Quantitative comparison with experiment is made and is generally satisfactory, but a lack of reliable data prevents definitive assessment of the general applicability of the model.

1. INTRODUCTION

The properties of superconductors under pressure have been the subject of numerous experimental and theoretical studies, especially since the advent of the BCS theory. Of particular interest is the behavior of the transition temperature $T_c$ under pressure. In spite of the successes of a number of theoretical studies of this problem, the phenomenon is still not well understood in general. The bulk of the work is summarized in a recent review.1

While the monotonic decrease of $T_c$ under pressure observed for most non-transition metals has been adequately described in several works with varying degrees of detail,2,3,4 the wide spectrum of pressure behavior observed amongst the transition metal superconductors remains largely unexplained. Indeed there is as yet no reliable basis on which to predict the sign of the pressure shift of $T_c$, much less its magnitude for the transition metal superconductors.

It has often been pointed out that the data required for accurate “first-principles” calculations of superconducting properties are generally not available. However, recent efforts in this direction by several authors5 have given reasonably accurate results for zero-pressure superconducting properties of simple metals. Among the transition-metal superconductors, ab initio calculations are not yet available, but the semiempirical approach initiated by McMillan6 and developed by Hopfield seems capable of accounting at least for the systematics of transition-metal superconductivity.

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In one instance where it has been possible to carry through a detailed calculation of the pressure shift of $T_c$, the results have been in excellent agreement with experiment.

In most cases, however, it is necessary to take certain superconducting data from experiment or to use unknown quantities as adjustable parameters in calculating other properties. While such an approach is highly useful for obtaining a description of the phenomena, it is not always easy to obtain from it any physical insight into the basic features of the problems. Therefore it may be helpful to study the phenomenon in a model system where the physical mechanisms operative are more transparent and the calculations require only basic properties independent of the superconducting state. This is the purpose of the present work.

The model used is essentially that studied by Morel and Anderson. We have chosen this particular starting point since it appears to be the most realistic model for which calculations can be readily made from a consistent theoretical basis. The idealizations in this model are numerous and have been discussed by Morel and Anderson, Ziman, De Gennes, and Seiden. We point out the principal approximations only briefly to indicate the extent to which the model may be expected to furnish quantitatively meaningful results. Firstly, the equation for the transition temperature is valid only in the weak-coupling limit, and in obtaining this result the phonon spectrum is approximated by an Einstein model and phonon enhancement effects are ignored. The coupling constant is calculated using the “jellium” approximation in which the effects of ion core size and of the crystal structure are ignored. Finally the contribution of normal scattering processes to the phonon-mediated interaction is virtually omitted for polyvalent metals.

Improvements on most of the above approximations can be and in fact have been made. The recent calculations by Seiden incorporate improvements on most of the above-noted approximations. Thus far however this prescription for the pressure dependence of $T_c$ has been applied only to certain nontransition superconductors, and experimental values of $T_c(V_0)$ and $\langle \partial \ln T_c/\partial \ln V \rangle_{V_0}$ are used to evaluate constants of the theory. The significance of the present work is then not its numerical result but its simplicity and the fact that it suggests an approach which may also be applicable to transition-metal superconductors.

2. MODEL CALCULATIONS

In the Morel-Anderson work the transition temperature is given by

$$T_c = c \theta_D \exp \left\{ -1/(\lambda - \mu^*) \right\}$$

where $c$ is a constant of order unity, and the coupling constant is given for $Z > 1$ by

$$\lambda = \frac{1}{4} \left[ \frac{a^2}{a^2 + \frac{3}{5}(4Z)^{-\frac{1}{2}}} \right]^2$$