IMPERFECT FLUID BD–FRW COSMOLOGIES

V. B. JOHRI and R. SUDHARSAN
Department of Mathematics, Indian Institute of Technology, Madras, India

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Abstract. A class of exact imperfect fluid solutions of the Brans–Dicke (BD) field equations, in which the geometrical part is identical to that of BD–FRW dust model with $k = 0$ is presented. The solutions are functions of time and radial coordinates and satisfy all necessary energy and thermodynamic conditions.

1. Introduction

In Einstein’s theory of general relativity (GR) the field equations relate the geometry of the space-time represented by the Einstein tensor $G_{ab}$ to the energy-momentum (EM) distribution represented by the physical tensor $T_{ab}$ through the coupling constant $8\pi G$ (with the velocity of light $c = 1$).

Tupper (1981, 1983a, b) has shown recently in relativistic cosmology, that given a space-time with the metric tensor $g_{ab}$, it is possible to find, in general, more than one EM distribution which satisfies all physical conditions and which equally represents the given space-time. Thus given the geometrical part of the solution $G_{ab}$, we do not necessarily have a unique physical part $T_{ab}$.

The different formulations of $T_{ab}$ for a given $G_{ab}$ can be obtained by the concept of tilted velocities. If a given space-time satisfies a perfect fluid distribution in a co-moving frame, then it is possible (Coley and Tupper, 1983a, b, 1985) to show that the space-time may also satisfy an imperfect fluid or an alternative fluid distribution in the same coordinate frame provided one is able to find a suitable velocity for the latter which is tilted with respect to the co-moving observers. To obtain a suitable velocity of tilt, one makes use of certain thermodynamic conditions.

The duality in the interpretation of $T_{ab}$ for a given $G_{ab}$ has been used by Goicoechea and Sanz (1984) to discuss the anisotropy in the cosmic microwave background radiation and also by others (Saha, 1983; Ferdon and Safko, 1985).

In this paper we have extended the concept of the tilted velocity used by Coley and Tupper in obtaining imperfect fluid solutions in GR to the BD theory*.

In the BD theory the physical part $T_{ab}$ consist of the EM distribution of matter and the long range scalar field generated by matter. The scalar field in the BD theory replaces...

* There is renewed interest in the BD theory after Mathiazhagan (Class Quantum Grav. 1, L29, 1984) showed that GR and BD theory behave very differently during inflationary era in the early universe; in fact, the BD theory might be more appropriate around GUT temperature as it allows the gravitational constant $G$ to vary fast analogous to other three basic interactions, although the observational constraints demanding $\omega > 500$ make the BD theory almost indistinguishable from GR in the matter dominated era.

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the reciprocal of the gravitational constant $G$ which couples matter distribution to the geometrical part in Einstein’s theory.

We have assumed that the given space-time, which is the $k = 0$, FRW metric corresponding to the dust distribution in the BD theory, admits an EM distribution which contains viscosity and heat flux and which generates a scalar field $\psi$. It is assumed that the scalar field $\psi$ generated by the imperfect fluid distribution varies as

$$\psi = A \phi^m,$$

(1.1)

where $\phi$ is the scalar field generated by the dust distribution, $A$ is a constant of proportionality, and the power index $m$ is a constant to be determined.

From the equivalence of the two distributions, namely the dust and the imperfect fluid along with their respective scalar fields, we find that $m$ takes values $1$ and $-3(\omega + 1)$.

In Section 2 the equations for imperfect fluid model are obtained. In Sections 3 and 4 we present two classes of solutions corresponding to the two values of $m$ and in Section 5 the results of Sections 3 and 4 are discussed.

### 2. The Imperfect Fluid Model

It is assumed that the given space-time in the BD theory admits two different EM distributions, dust and imperfect, with two different scalar fields $\phi$ and $\psi$, respectively, which are functions of time alone.

If $G_{ab}$ represents the geometrical part, in the BD theory, we have the following equivalence relation between the two distributions:

$$\frac{8\pi}{\psi} M_{ab} + F_{ab}(\psi) = G_{ab} = -\frac{8\pi}{\phi} T_{ab} + F_{ab}(\phi),$$

(2.1)

where

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R$$

(2.2)

is given by

$$ds^2 = -dt^2 + R^2(t) [d\theta^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2];$$

(2.3)

and

$$F_{ab}(\phi) \equiv -\frac{\omega}{\phi^2} [\phi_{,a} \phi_{,b} - \frac{1}{2} g_{ab} \phi^c \phi_{,c}] - \frac{1}{\phi} [\phi_{,a} \phi_{,b} - g_{ab} \Box^2 \phi],$$

(2.4)

along with

$$\Box^2 \phi = \frac{8\pi}{3 + 2\omega} T^c_c,$$

(2.5)

and

$$\Box^2 \psi = \frac{8\pi}{3 + 2\omega} M^c_c;$$

(2.6)