HYDRODYNAMICAL MODES AND GRAVITATIONAL INSTABILITIES IN THE UNIVERSE DURING THE RECOMBINATION ERA

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Abstract. We assume the Universe during the recombination era as a three-component fluid constituted by neutral hydrogen, plasma, and radiation; such fluids are coupled via the effects of photorecombination, photoionization, and Thomson scattering. The hydrodynamical modes are calculated and the relation with the gravitational instabilities is established. In addition to the well-known Jeans's instability modes two additional ones in the neighbourhood of $10^{13} M_\odot$ are obtained in the case of an open Universe.

1. Introduction

The Jeans treatment of the perturbations in an homogeneous fluid was a classical (i.e., a Newtonian) one. A thorough investigation of the theory of perturbations in the framework of general relativity theory was undertaken by Lifshitz (1946), Lifshitz and Khalatnikov (1963), Hawking (1966), Harrison (1967), and Jones (1976). Developing further this type of ideas about the treatment of perturbations, we will consider the Universe in the recombination era and treat the matter substratum, as a three-component fluid consisting of neutral hydrogen, hydrogen plasma, and radiation. We apply the theory of perturbations to this 3-component gas with the intention to obtain an initial spectrum of fluctuations, which could be the progenitors of galaxies. We take into account the interactions between the 3 components in the following way: radiation and plasma interact via Thomson scattering; the hydrogen gas has interactions with the radiation via the process of photoionization and with the plasma via photorecombination. The pressure of each gas-component will be taken into account as that of an ideal gas. We will seek solutions of the characteristic hydrodynamic equations of our model, neglecting for simplicity in a first step the expansions of the Universe.

The significance of our work is that it is the first model of the Universe, where hydrogen, plasma, radiation and their interactions are considered. Up to now only 2-component fluid models have been investigated; namely, by Peebles and Yu (1970), Nowotny, (1980, 1981), Hsieh (1975), Hsieh and Spiegel (1976), Silk (1968), Weinberg (1971), Bonometto and Lucchin (1975, 1976, 1977), Meszáros (1979), and others.

We begin with a discussion of the equations of state. We use the relativistic theory of perturbations to derive the equations of the fluid, i.e., equations of conservation of...
mass, the momentum-balance equations and energy-balance equations for each component of the fluid. The gravitational field equation is written down in synchronous gauge (see, e.g., Weinberg, 1972).

In Section 3 we solve the equations of motion. We discuss the numerical solutions in Section 4. The Jeans mode shows a very similar behaviour with that obtained by Nowotny (1980) in a two-fluid analysis. However, two growing oscillating modes are also present, and the fact that these modes become unstable near $10^{12} M_\odot - 10^{14} M_\odot$ can be interpreted as a privileged region, meaning that structures with very high masses have originated prior to other cosmological structures.

In this work, greek letter run from 0 to 3 and latin letters from 1 to 3. The covariant derivative is denoted by '||', the partial derivative by '!' and $g^{\mu\nu}$ has signature + 2.

2. The Basic Equations

We assume that the fluid obeys the equation of state of an ideal gas: i.e., that

$$p_\alpha = \frac{N_\alpha k_B T_\alpha}{V}, \quad (2.1)$$

where $k_B$ represents Boltzmann's constant, $T_\alpha$ the temperature, and $N_\alpha$ the number of particles of the $\alpha$-component of the fluid (hydrogen $\alpha = g$, and plasma $\alpha = p$, radiation $\alpha = r$).

That equation for hydrogen can be written as

$$p_g = \frac{\rho_g k_B T_g}{m_H}, \quad (2.2)$$

and the caloric equation is

$$\varepsilon_g = \frac{3 \rho_g k_B T_g}{2m_H}; \quad (2.3)$$

$\varepsilon_g$ representing the internal thermal energy of the hydrogen gas, $\rho_g$ the mass density of the hydrogen gas, and $m_H$ the mass of the hydrogen atom.

The equation of state for the plasma will be

$$p_p = \frac{2\rho_p k_B T_p}{m_H}, \quad (2.4)$$

where $\rho_p$ is the mass density of the plasma gas. The corresponding caloric equation is

$$\varepsilon_p = \frac{3\rho_p + \chi \rho_p}{2m_H}, \quad (2.5)$$

in which $\varepsilon_p$ represents the internal thermal energy of the plasma gas and $\chi$ the ionization potential of the hydrogen atom in the ground state.