DISTRIBUTION FUNCTIONS IN THE STATISTICAL THEORY OF MHD TURBULENCE OF AN INCOMPRESSIBLE FLUID IN THE PRESENCE OF THE CORIOLIS FORCE

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Abstract. In this paper we have made an attempt to define hierarchy of distribution functions for the simultaneous velocity and magnetic fields. Various properties of the constructed functions such as reduction property, separation property, coincidence properties have been discussed, and equations for the evaluation of one- and two-point bivariate distribution functions have been derived.

1. Introduction

Nowadays, two major and distinct areas of investigations in non-equilibrium statistical mechanics are the kinetic theory of gases and statistical theory of fluid turbulence. Hopf (1952), Kraichnan (1969), Edwards (1964), and Herring (1964) have significantly contributed to this theory. Further attempts were made by Lundgren (1967). He derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions which resemble with BBGKY hierarchy of equations in the kinetic theory of gases. But, at this stage, one is met with the difficulty that n-point distribution function depends on (n + 1)-point distribution function and thus results in an unclosed system. This so-called closure problem is encountered in the kinetic theory of turbulence and other nonlinear systems. In this paper an attempt is made to define the distribution function for the simultaneous velocity and magnetic field in MHD turbulence in the presence of the Coriolis force. It is the extension of work done by Lundgren (1967).

2. Basic Equations

The equations governing the viscous, incompressible MHD turbulent flows as given by Chandrasekhar (1956) are:

$$\frac{\partial v_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (v_\alpha v_\beta - h_\alpha h_\beta) = - \frac{\partial w}{\partial x_\alpha} + v \nabla^2 v_\alpha - 2 \epsilon_{\alpha\beta\gamma} \Omega_\gamma v_\alpha ,$$ (2.1)

$$\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (h_\alpha v_\beta - v_\alpha h_\beta) = \lambda \nabla^2 h_\alpha ,$$ (2.2)

$$\frac{\partial v_\alpha}{\partial x_\alpha} = 0 ,$$ (2.3)
\[
\frac{\partial h_\alpha}{\partial x_\alpha} = 0; \tag{2.4}
\]

where

\[
w = \frac{p}{\rho} + \frac{1}{2} |h|^2 + \frac{1}{2} |\Omega \times x|^2
\]

stands for the generalized pressure \( \lambda = \left(4 \pi \mu_c \sigma \right)^{-1} \) is magnetic diffusivity, \( \Omega \) the angular velocity vector of a uniform rotation. The repeated suffices are assumed over the values 1, 2, and 3 and unrepeated suffices may take any of these values.

The total pressure \( w \) occurring in Equation (2.1) may be eliminated by taking the divergence of Equation (2.1). Also in a conducting infinite fluid, only the particular solution of the resulting equation is relevant; and so we have

\[
w = \frac{1}{4\pi} \int \left[ \frac{\partial v'_\alpha}{\partial x'_\beta} \frac{\partial v'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta} \frac{\partial h'_\beta}{\partial x'_\alpha} \right] \frac{dx'}{|x' - \bar{x}|}. \tag{2.5}
\]

Hence, Equations (2.1) and (2.2) become

\[
\frac{\partial v_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (v_\alpha v_\beta - h_\alpha h_\beta) = -\frac{1}{4\pi} \frac{\partial}{\partial x_\alpha} \int \left[ \frac{\partial v'_\beta}{\partial x'_\alpha} \frac{\partial v'_\alpha}{\partial x'_\beta} - \frac{\partial h'_\beta}{\partial x'_\alpha} \frac{\partial h'_\alpha}{\partial x'_\beta} \right] \frac{dx'}{|x' - \bar{x}|} + v \nabla^2 v_\alpha - 2\eta \frac{\partial}{\partial x_\alpha} \Omega_{\beta\alpha} v_\beta, \tag{2.6}
\]

\[
\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (h_\alpha v_\beta - v_\alpha h_\beta) = \lambda \nabla^2 h_\alpha. \tag{2.7}
\]

3. Formulation of the Problem

We consider large identical fluids, each member being an infinite incompressible conducting fluid in turbulent state. No external electric or magnetic field is used to supply the electromagnetic energy in the flow field, but it arises only due to hydrodynamical motion. The fluid and Alfvén velocities \( \vec{v} \) and \( \vec{h} \) are randomly distributed functions of position and time and satisfy the equations of motion and continuity given by (2.1)–(2.4). Different members of ensemble are subjected to different initial conditions, and our aim is to find out a way by which we can determine the ensemble averages at the initial time. Certain microscopic properties of conducting fluids, such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time, can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). Our present