GRAVITATIONAL COLLAPSE OF A MASSIVE SPHERE OF CONSTANT ENERGY DENSITY

(Letter to the Editor)

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Abstract. Gravitational collapse of a massive sphere of constant density has been studied from the point of view of a Keplerian observer. The asymptotic nature of collapse is attributed to the development of negative gravitational pressure acting radially outwards within the structure. The region of negative pressure asymptotically covers the entire interior as \( u = \text{mass/radius} \) tends to half.

1. Introduction

A stellar structure remains in hydrostatic equilibrium till the gravitational pull (pressure) is balanced by the material pressure. During the evolution a stage comes when the gravitational pressure overpowers the material pressure and the collapse sets in. For a structure with a constant energy density, when \( u = \frac{4}{3} \), the gravitational pressure becomes infinite at the centre of the structure, and no material pressure can stop the gravitational collapse. However, the story does not end up at \( u = \frac{4}{3} \). The collapse proceeds asymptotically and \( u \rightarrow 0.5 \) as \( t \rightarrow \infty \). In this paper we have tried to understand the physical reason for the asymptotic behaviour of the collapse. It is shown that as the collapse proceeds and \( u \) becomes greater than \( \frac{4}{3} \), the gravitational pressure in one region tries to retard the process of collapse in such a way that an external Keplerian observer observes the collapse as an asymptotic process taking an infinite time.

The Keplerian observer gets more and more redshifted signals from the object as the collapse proceeds. This redshift gives the value of \( u \); and the observer if he is a relativist can calculate the metric coefficients and the gravitational pressure granting that the structure is of uniform density and that it maintains uniform density throughout the collapse.

We first calculate the value of \( e^\nu \) and pressure at various points within the structure for different \( u \)-values and picturize the scenario of collapse. For a massive sphere of constant energy density \( E \), the field equations can be solved exactly (Schwarzschild, 1916; Tolman, 1939; Weinberg, 1972). In this case, for a mass to size ratio \( u = \frac{4}{3} \), the metric parameter \( e^\nu \) increases (with a value \( \frac{1}{3} [3(1 - 2u) - 1]^2 \) at the centre) as one moves towards the surface of the structure. In terms of the pressure \( P \) and \( e^\nu \) we have the relation:

\[
P = E(R - 1) \quad \text{with} \quad R = e^{-(\nu - \nu_0)/2}.
\]
This means that \( P \) is directed inwards and decreases towards the surface (at \( r = a, P = 0 \)).

When the structure undergoes a gravitational collapse, i.e., it becomes more and more compact, the \( \upsilon \)-value increases. At \( t = \frac{3}{4}, \upsilon = 0 \) (\( \upsilon \) at \( r = 0 \)) becomes zero and the central pressure is infinite. Further collapse implies a \( \upsilon \)-value higher than \( \frac{4}{3} \) and an altogether different behaviour of \( \upsilon \) and \( P \) within the structure.

The subject matter of this paper is a detailed study of the variation of \( \upsilon \) and \( P \) with \( y(\gamma = a) \) for \( \frac{4}{3} < \upsilon < \frac{1}{2} \) and a discussion of scenario from the point of view of an external Keplerian observer.

2. Variation of \( \upsilon \) and \( P \)

For a spherically-symmetrical mass of constant energy density one has (Durgapal and Pande, 1979)

\[
\upsilon = \frac{1}{3} \left[ 3(1 - 2\upsilon)^{1/2} - (1 - 2\upsilon y^2)^{1/2} \right]^2.
\]  

(2)

For \( \upsilon < \frac{4}{3} \), \( \upsilon \) takes its minimum value at the centre (\( \upsilon = 0 \)), and with increasing \( y \) it increases to a value (1 - 2\upsilon) at the surface (\( \upsilon = 1 \)). When \( \upsilon \) equals \( \frac{4}{3} \), \( \upsilon \) is vanishing and the surface value \( \upsilon \) is \( \frac{1}{2} \). For \( \frac{4}{3} < \upsilon < \frac{1}{2}, \upsilon > 0 \); and with increasing of \( y \) it decreases becoming vanishing at some \( y \)-value (\( y = y_\infty \)); with further increase of \( y \), \( \upsilon \) increases and finally becomes (1 - 2\upsilon) at the surface. As \( \upsilon \) is increased beyond the value \( \frac{4}{3} \), \( y \) also increases and approaches 1 as \( \upsilon \) approaches the black-hole limit \( \frac{1}{2} \). This is shown in

![Fig. 1. \( y_\infty \) vs \( \upsilon \) (\( \epsilon \) = const.).](image-url)