THE EFFECTS OF LARGE-SCALE MOTION ON THE REGENERATION OF TURBULENT DYNAMO

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Abstract. The kinematic turbulent dynamo equations are studied in the presence of a large-scale velocity field. The two length-scales approach is employed and solutions of the equations are found in the limit of small bulk motion and shear, and for large Reynolds number $R_m$. The regeneration term is calculated up to second-order in $1/R_m$ using cyclonic convective turbulent velocity field.

1. Introduction

In the past three decades the kinematic theory of turbulent dynamo proposed by Parker (1955) has been extensively studied. It is capable to explain how astrophysical magnetic fields are generated and sustained. Several different approximations and simplifications have been employed in studying the dynamo equations which are quite difficult to solve in their most general form. For instance, almost every work uses the 'first-order smoothing' approximation (Moffatt, 1970, 1974; Lerche, 1971b; Levy, 1978) which neglects terms containing products of the turbulent component of the velocity and magnetic field.

Two other limits are commonly considered: in a high conducting medium the magnetic Reynolds number may be taken as infinite and the diffusion can be neglected. The magnetic field lines move together with the velocity of the fluid (Parker, 1955, 1970). The other limit is for a poorly conducting medium, where the magnetic Reynolds number is low. In this case the field variation is slow and time derivatives are negligible compared with the diffusion term (Moffatt, 1970; Lerche, 1971b). Some improvement can be made to these limits. For instance, Levy (1978) calculated the regeneration term of the dynamo equation for finite, high magnetic Reynolds number in the particular case of the cyclonic convective eddies introduced by Parker (1970). More recently, Carvalho and Pires (1986) used the same velocity field to calculate a general solution for the turbulent component of the velocity and magnetic field. Their results are identical to those of Parker and Levy in the asymptotic limit.

Besides from the approximations mentioned above, most works consider only a small-scale turbulent velocity field, neglecting any bulk velocity or velocity shear. The inclusion of a large-scale velocity field makes the dynamo equations quite complex. It is only when one considers small bulk velocity and shear that the mathematical difficulties can be overcome. This problem has been subject of consideration of some authors (see, e.g., Lerche, 1971a, b; Krause and Roberts, 1973) although no concrete
example has yet been worked out. For instance, Parker (1970) discussed the effect of velocity shear on the large-scale component of the magnetic field. However, in calculating the regeneration term using his cyclonic velocity eddies, bulk shear or bulk motion is not taken into account. As we show below this is justified only in the case when the large-scale velocity is small and the conductivity is infinite. Other specific calculations of the regeneration term (Levy, 1978; Carvalho and Pires, 1986) also neglect large-scale motion.

The purpose of this work is to study the effect of bulk motion on the regeneration term of turbulent dynamo. To keep mathematical complexity to a minimum we shall employ the high magnetic Reynolds number approximation and, in order to allow comparison with early work, we use the cyclonic convective turbulent cells of Parker (1970). In the next section we derive the dynamo equation for the turbulent component of the magnetic field in the presence of a large-scale velocity in the limit of small resistivity. In Section 3 we discuss the results of the calculation of the regeneration term and in Section 4 we give the main conclusions.

2. The Effect of a Small Bulk Motion

The behaviour of magnetic fields in an electrically-conducting fluid is described by the hydromagnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} - D \nabla^2 \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (1)$$

where \( \mathbf{B} \) is the magnetic field; \( \mathbf{V} \), the fluid velocity and the magnetic diffusivity is \( D = \frac{c^2}{4\pi\sigma} \), with \( \sigma \) the electrical conductivity. Suppose that both the magnetic and velocity field can be separated into a large-scale and a small-scale field such that

$$\mathbf{B} = \mathbf{B}_o + \mathbf{b} \quad \text{and} \quad \mathbf{V} = \mathbf{V}_o + \mathbf{v}; \quad (2)$$

where \( \mathbf{B}_o \) and \( \mathbf{V}_o \) are the large-scale component and \( \mathbf{b} \) and \( \mathbf{v} \) the turbulent component of the magnetic and velocity field, respectively. It is assumed that the averages \( \langle \mathbf{b} \rangle \) and \( \langle \mathbf{v} \rangle \) vanish and that the mean fields \( \mathbf{B}_o \) and \( \mathbf{V}_o \) are essentially constant over the small-scale \( l \) of variation of \( \mathbf{v} \) but they vary on a large-scale \( \mathcal{L} \gg l \).

The ensemble average of Equation (1) gives

$$\frac{\partial \mathbf{B}_o}{\partial t} - D \nabla^2 \mathbf{B}_o = \nabla \times (\mathbf{V}_o \times \mathbf{B}_o) + \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle. \quad (3)$$

If we subtract Equation (3) from Equation (1) we obtain

$$\frac{\partial \mathbf{b}}{\partial t} - D \nabla^2 \mathbf{b} = \nabla \times (\mathbf{V}_o \times \mathbf{b}) + \nabla \times (\mathbf{v} \times \mathbf{B}_o), \quad (4)$$

where we have used the sual smoothing approximation and neglected the term \( \nabla \times [(\mathbf{v} \times \mathbf{b}) - \langle \mathbf{v} \times \mathbf{b} \rangle] \). The solution of Equation (4) for \( \mathbf{b} \) together with \( \mathbf{v} \) can be used