EFFECT OF AZIMUTHAL MAGNETIC FIELD ON THE PROPAGATION OF CYLINDRICAL SHOCK WAVES IN SELF-GRAVITATING GAS

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Abstract. A propagation of diverging cylindrical shock in a self-gravitating gas, having an initial density and azimuthal magnetic field distributions variable, has been studied for the two cases (i) when the shock is weak and (ii) when it is strong. Analytical relations for shock velocity and shock strength have been obtained. Lastly, the expressions for the pressure, the density and the particle velocity immediately behind the shock have been also obtained for both cases.

1. Introduction

The study of the propagation of hydromagnetic shocks is relevant to the phenomenon of thunder. Pai (1958, 1959) and Kumar et al. (1981) have investigated the propagation of hydromagnetic-cylindrical shock waves through a self-gravitating gas which are valid for strong shocks only. Kumar et al. (1982) and Kumar and Prakash (1982) have used the CCW method to investigate the propagation of diverging cylindrical shock waves in an ideal gas in the presence of an axial magnetic field for both weak and strong shocks. Kumar and Saxena (1984) solve the same problem when density obeys the power law. They have taken a constant axial component of the magnetic field.

In the present paper the CCW (cf. Chester, 1954; Chisnell, 1955; Whitham, 1958) method has been used to study the propagation of diverging hydromagnetic-cylindrical shock waves through an electrically conducting self-gravitating gas of infinite conductivity, having an initial density and azimuthal magnetic field distributions, respectively, \( \rho_0 = \rho_c r^{-w} \) and \( H_0 = H_c r^{-w} \), simultaneously for the two cases; (i) when the shock is weak and (ii) when it is strong. Thus we have taken both density and magnetic field variable and azimuthal magnetic field in place of axial component of the magnetic field because it is more effective. The case of weak shock is explored under two conditions: (i) when the magnetic field is weak and (ii) when it is strong. Similarly, the analytical expressions for shock velocity and shock strength have been obtained for strong shock under two conditions: (i) when the ratio of densities on either side of the shock nearly equals \((\gamma + 1)/(\gamma - 1)\); and (ii) when the magnetic field is strong.

Finally, the expressions for the pressure, the density and velocity just behind the shock have been derived also for both the cases.
2. Basic Equations, Boundary Conditions, and Analytical Expressions for Shock Velocity

The equations governing the cylindrically-symmetrical flow of gas under the influence of its own gravitation in the presence of transverse magnetic field, following Whitham (1958) and Sedov (1982), are:

\[
\begin{align*}
d\frac{u}{dt} & \left( \frac{1}{r} \right) + \frac{1}{r^2} \frac{d}{dr} \left( r \frac{du}{dr} \right) + \frac{\partial}{\partial r} \left( \frac{1}{\rho} \right) \frac{dp}{dr} + \frac{Gm}{r} \frac{\partial H}{\partial r} + \frac{\mu H^2}{\rho r} = 0, \\
\frac{\partial p}{\partial t} & + \frac{u}{\rho} \left( \frac{\partial p}{\partial r} \right) + r \left( \frac{1}{\rho} \right) \frac{du}{dr} = 0, \\
\frac{\partial p}{\partial t} & - a^2 \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial r} \right) = 0, \\
\frac{\partial m}{\partial r} - 2\pi \rho \frac{d}{dr} & + \frac{\partial H}{\partial t} + u \frac{\partial H}{\partial r} + H \frac{\partial u}{\partial r} = 0;
\end{align*}
\]

where \( m(r, t), u(r, t), p(r, t), \rho(r, t), \) and \( H(r, t) \) denote, respectively, the mass inside a cylinder of radius \( r \), the velocity, the pressure, the density, and the transverse magnetic field at a distance \( r \) at time \( t \). \( \mu \) is the magnetic permeability and \( a^2 = \gamma \rho / \rho \).

The magnetohydrodynamic conditions can be written in terms of a single parameter \( N = \rho_1 / \rho_0 \) as

\[
\begin{align*}
\rho_1 &= N \rho_0, \\
H_1 &= NH_0, \\
u_1 &= \left( 1 - \frac{1}{N} \right) U, \\
U^2 &= \frac{2N}{(\gamma + 1) - (\gamma - 1)N} \left[ a_0^2 + b_0^2 \left( 2 - \gamma \right) N + \gamma \right], \\
p_1 &= p_0 + \frac{2\rho_0(N - 1)}{(\gamma + 1) - (\gamma - 1)N} \left\{ a_0^2 + \frac{\gamma - 1}{4} b_0^2(N - 1)^2 \right\};
\end{align*}
\]

where \( 0 \) and \( 1 \), respectively, stand for the states just ahead and just behind the shock front; \( U \) is the shock velocity, \( a_0 \) is the sound speed \( (\gamma p_0 / \rho_0)^{1/2} \), and \( b_0 \) is the Alfvén speed \( (\mu H_0^2 / \rho_0)^{1/2} \).

2.1. Weak Shocks

For very weak shock we take the parameter as

\[
\frac{\rho_1}{\rho_0} = N = 1 + \varepsilon,
\]

where \( \varepsilon \ll 1 \). Now consider the two cases of weak and strong magnetic fields.