EQUATIONS OF RADIATION GAS DYNAMICS IN TETRAD FORM FOR ASTROPHYSICAL APPLICATIONS

YU. I. MOROZOV

Institute of Experimental and Theoretical Physics, USSR Academy of Sciences, Moscow, U.S.S.R.

(Received 10 November, 1986)

Abstract. This paper deals with the representation of relativistic equations of gas dynamics with due regard to the general relativity theory effects in the form accepted and widely applied in the special relativity theory. With this purpose, a strict formal definition of a non-inertial co-moving reference frame without rotation is carried out on the basis of a tetrad formalism by use of the Fermi-Walker rules of transport of 4-frame. The equations of physical kinetics, relativistic collapse, Einstein's equations, equations of relativistic radiation gas dynamics for ideal and dissipative gases, Taub's equations for a shock wave, which allow for radiation and electron-positron pairs, are obtained in this reference frame. On the basis of the local Lorentz transformation and the Ricci rotation coefficients, these equations are written in the laboratory reference frame, in order to illustrate the fact that the general relativity effects can be simply taken into account in the equations having a form accepted in the special relativity theory.

1. Introduction

Nowadays an ever increasing attention is paid to studies of the phenomena characterized by relativistic velocities and high gas temperatures, by a great significance of gravitation effects and non-inertial character of the motion of matter, by the presence of discontinuities as well as by a great contribution of equilibrium and non-equilibrium radiation in the motion dynamics. In this respect one could mention, for example, the problems of supernova outbursts, gravitational collapse, solar wind, cosmic rays, in which the listed effects play a very important, if not determining, role. The consideration of these effects is not simple, and requires not only application of specific approaches and various methods of description, but, frequently, the physical substantiation of possibilities of using them. For example, there may be various approaches to inclusion of the general relativity (GR) effects in the relativistic gas dynamics (RGD). The approach is popular, in which the heuristic principle of general covariancy, developed effectively by Einstein, is used. However, this principle is often implemented physically on the basis of a point set of locally-inertial reference frames falling freely in a curved space-time. Thus, such an important process – from the viewpoint of substantiation of applicability of kinetics principles – as mutual collision of mass particles, or collision between particles and photons, will be considered. In fact, in two different frames corresponding to the beginning and to the end of a collision act and, hence, the momentum and energy of interacting particles, will vary in the intervals between collisions. Meanwhile, one of the assumptions underlying the possibility of using the kinetic consideration, consists just in the absence of such a variation (see, for example, the monograph by De Groot et al., 1980). This fact is especially important for the consideration of interaction between photons and matter. Therefore, the use of this
approach for including the GR effects into the radiation relativistic gas dynamics (RRGD) assumes the consideration of gravity as a long-range force to which photons are subordinated between collision acts. Such an approach is used, for example, by Chernikov (1962). The approach which uses a non-inertial co-moving frame, moving in a curved space-time under an influence of non-gravitational forces (for example, gaseous pressure or electromagnetic forces), has certain physical advantages. First, since this system implements the physical laboratory situation (which involves both initial and final collision points) the requirement that the energy-momentum of colliding particles does not vary between collision acts, strictly holds good. As a result, the phase trajectory of the system under study in a seven-dimensional space \{t, \vec{x}, \vec{p}\} is reduced to usual trajectory in a configurational space \{t, \vec{x}\}, which considerably simplifies the physical kinetics equations. Second, such an approach may substantiate the use of RRGD whenever the shock wave are present – especially for situations with a great role of radiation, in which the discontinuities are essentially ‘washed down’. In any case, one may avoid an ambiguity in defining the co-moving frame, which is mentioned in the paper by Smarr et al. (1979), either by use of the equations incorporating heat conduction and viscosity (including those for radiation), or by including into consideration (in any manner) the artificial viscosity which is generally needed in numerical calculations (see Roache, 1976). Third, the use of co-moving frames is substantiated from a physical viewpoint to a large extent, since these systems in which the parameters, related to observed values, are defined. Fourth, the co-moving frame may be defined in such a way that the local Lorentz transformations are valid, which allows not only to use effectively the principle of correspondence with the special relativity (SR), but also to represent a number of GR effects in a form used in SR. This latter aspect is especially important, since many useful developments in the field of co-moving reference frames in GR have acquired such a specific character that the obtained results are, in fact, inaccessible for many gas-dynamic specialists who got used to classical RGD equations. Thus, the purpose of this study consists mainly in transforming some important results, obtained on the basis of a strict application of one of the variants of a tetrad formalism, to the form mostly known in SR, so that gas dynamic specialists could include some important GR and RRGD effects into their studies with least difficulty, and with a maximum degree of validity.

Let us consider the sources and parallels of the approach proposed. We shall propose the definition of a co-moving reference frame based on the transport of 4-frame along the coordinate curves in accordance with the rules, which had been first formulated for the case of transport along the world line by Walker (1932) and Fermi (1972). Such a space-like transport from the viewpoint of a correct definition of ‘physical laboratory’ had been considered by Synge (1960) and Moeller (1972) on the basis of a formal technique described in Eisenhart’s book (Eisenhart, 1926). One may note that such a definition with describing the possibility of its propagation to quantum mechanic problems was first suggest apparently by Thomas (1945). He had also formulated correctly the RRGD equations within the SR framework (Thomas, 1930). These equations have been further developed by Synge (1957), Misner and Sharp (1964),