THE AREAS OF MOTION IN THE PLANAR MAGNETIC-BINARIES PROBLEM

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Abstract. The purpose of this article is to present the areas of motion in the planar magnetic-binaries problem and show their contribution to the behavioral study of a charged particle moving in such a system.

1. Introduction

The foundation of the magnetic-binaries problem, established in a recent article (Mavraganis, 1978), is based on the equations of classical mechanics. The proper restrictions of the problem are those of the circular motion of the primaries around their center of mass and of the axial symmetry of the magnetic fields (using the dipole representation) related to the primaries.* The case of the planar motion is a convenient point at which to study the general behavior of a particle in such a system. Thus, using the (dimensionless) synodic coordinate system, we investigate the areas of planar motion, having as a tool the zero-velocity curves. Much useful information can be obtained as a result of this work – for example, looking for special types of motion, stability, branchings of the family, etc.

2. Zero-Velocity Curves

The planar motion of a charged particle in a system of magnetic binaries is tested by the integral (energy) equation

\[
\frac{1}{2}(x^2 + y^2) = \frac{1}{2}(x^2 + y^2) + (x^2 + y^2)p_{1\gamma} - xq_{1\gamma} - C, \tag{1}
\]

where

\[
\begin{align*}
   p_{1\gamma} &= \frac{\gamma_1}{r_1^3} + \frac{\lambda_2 y_2}{r_2^3}, \\
   q_{1\gamma} &= \frac{\mu_2 y_2}{r_1^3} - \frac{\lambda_1 y_2}{r_2^3}, \\
   r_1^2 &= (x - \mu_2)^2 + y^2, \\
   r_2^2 &= (x + \mu_1)^2 + y^2, \\
   \mu_1 &= 1 - \mu, \\
   \mu_2 &= \mu \quad (\mu \text{ is the reduced mass})
\end{align*}
\]

* All of these are expressed by eight parameters, the ratio and the directional cosines of the magnetic moments and the mass of the primaries.

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and \( \lambda \) and \( \gamma \) are the ratio and the directional cosines of the magnetic moments, related to the primaries, respectively. This integral enables us to investigate the regions of the plane \( z = 0 \) in which the particle is permitted to move, using for this purpose the zero-velocity curves. These curves are given by the function

\[
f(x, y; C) = (x^2 + y^2) + 2(x^2 + y^2)p_{1\gamma} - 2xq_{1\gamma} - 2C
\]

(3)

for several values of the constant \( C \) and limit motion within certain parts of the plane. The particle arriving at the boundary of these parts acquires a zero velocity and is then reflected back, perpendicularly to the boundary, which corresponds to the same constant \( C \) of the motion (Szebehely, 1967).

The permissible regions of the motion are in some cases closed, 'trapping' the particle inside them. This fact leads to the conjecture of the existence of Van Allen zones, because, as is known, these zones are located in the closed areas (of motion) of the Earth's magnetic field. Assuming that the hypothesis of Poincaré (1892) on the density of the solutions (as tested by Katsiaris and Goudas (1972)) is valid in the magnetic-binary problem, we expect much information about the general behavior of the particle by studying special types of motion – periodic motion, almost periodic motion, symmetric periodic motion, etc. – in the permissible areas and properly in the closed areas.

Since the system governing the planar motion is autonomous, we can write again the integral (1) using the initial conditions \( x_0 = (x_0, y_0, x_0, y_0) \) of the solutions: namely,

\[
x_0^2 + y_0^2 = f(x_0, y_0; C) \geq 0;
\]

(4)

or, putting \( y_0 = 0 \) (without a loss of generality),

\[
x_0^2 + y_0^2 = g(x_0; C) \geq 0.
\]

(5)

This last relation gives a curve in the plane \( (x_0, C) \), which corresponds to zero velocity of the particle and separates the permissible area of motion from the forbidden area. This curve contributes greatly to the study of planar motion, because it permits us to investigate special types of motion (e.g., symmetric periodic motion) and thus gives a simpler view of the areas of motion. As a result, we immediately notice that two lines exist at the points \( x = \mu_2 \) and \( x = -\mu_1 \), which lead the curve to infinity with a dense pass of the zero-velocity curves through these points. On the other hand, we see the relative extrema of the function \( g(x_0; C) \); that is to say, equal in number, equilibria points of the motion.

3. Numerical Investigation

The numerical investigation of the zero-velocity curves is divided into two phases. In Phase I the values of the constant \( C \) are carefully chosen by means of the relation