SOME NONLINEAR EFFECTS IN THE RELATIVISTIC TWO-BODY PROBLEM

K. A. PYRAGAS, V. I. ZHDANOV, A. N. ALEXANDROV and L. E. PYRAGAS
State Standards Committee, Moscow, U.S.S.R.

(Received 20 January, 1978)

Abstract. The test-particle motion in the centrally symmetric gravitational field can be described by the equation in the form appropriate for a nonlinear oscillator – the nonlinear terms being due to the nonrelativistic effects. This enables us to apply to this equation the well-known asymptotic methods of the theory of nonlinear oscillations. Typical nonlinear oscillation phenomena arising from the action of external forces are shown to take place. The form of equations and the main results remain valid in the problem of two bodies of comparable mass in the post-Newtonian approximation.

1. Introduction

The nonlinearity of field equations in Einstein's theory of gravitation is certainly one of its most significant features. However, the nonlinearity of field equations is primarily responsible for a number of mathematical and physical difficulties encountered in this theory. In particular, the absence of appropriate mathematical methods for analyzing nonlinear systems in partial derivatives does not permit an exact solution or a strict qualitative analysis of even the simplest problems of gravitating systems (for example, the problem of two bodies of comparable mass) within the framework of general relativity (GR). The presently available methods of obtaining exact solutions of the field equations, based on the symmetry hypothesis or the algebraic structure of a curvature tensor (Petrov, 1965) encounter considerable difficulties when the results are interpreted physically; and also give little information on the nonlinear character of the interaction of gravitating systems. Certain progress in the analysis of some classes of nonlinear systems with distributed parameters – made recently by studying solitons – has not yet produced any appreciable results in GR. Physically, the nonlinearity of field equations results in the well-known difficulties hindering the construction of the quantum theory of gravitation, the construction of conserved quantities, the theory of gravitational radiation, etc.

On the other hand, it is known that nonlinear systems differ from linear systems in a number of important features. These differences became pronounced after the development of methods which enabled us to analyse nonlinear systems involving a small parameter (cf. Bogolyubov and Mitropolsky, 1974), as well as study a number of essentially nonlinear equations in partial derivatives. The well-known property of
nonlinear systems to conserve certain characteristics invariable under the long action of external forces seems to play a very important role in nature, and must have numerous consequences in relativistic astrophysics and cosmology. The exceptional complexity of the resulting dynamical systems does not, however, promise rapid progress in their investigation. This holds especially true for the problems requiring field-theoretical treatment. Some nonlinear effects of the gravitational interaction can, however, be studied without employing field-theoretical treatment, which considerably reduces the mathematical difficulties. This refers to the problems concerned with the motion of test bodies in predetermined gravitational fields and to the cases when relativistic corrections to the equations of motion are small and it is possible to use the post-Newtonian approximation (PNA) of GR. Certainly, this approach avoids many important problems in GR such as the motion of two black holes, etc., and it can be regarded as preliminary; but still it allows one to elucidate certain aspects of the nonlinearity of the interaction.

In the present paper we study some nonlinear effects in the problem of two gravitating bodies under the following assumptions: (a) one of the bodies is heavy, and the other is a test body, but its influence on the heavy body can be neglected; (b) both bodies possess comparable masses, but the relativistic corrections are small and their motion is described by the PNA of GR. This simplified problem can find application in the theory of double stars; it can also be used to study the accretion of matter into a black hole. The essential feature of the present paper is that we consider the influence of some phenomenological factors (low-energy dissipation and external forces on the motion of the system in question. The asymptotic methods in the theory of nonlinear oscillations (Bogolyubov and Mitropolsky, 1974) permit a detailed analysis of these problems. The conclusions derived may also be useful for estimating the flow of gravitational radiation from double stars.

2. The Basic Equations and their Analysis

Consider the relative motion of two bodies, one of which is a reference body and the other a test body. It is known that if a test body moves along the geodesic line, and if the supporting trajectory is also geodesic, the equations of the relative motion to the first approximation are the equations of geodesic deviation. The equations of motion to the second approximation relative to the parameters of deviation were obtained by Bazhanski (1974). Similar results were also obtained by Hodgkinson (1972), who has considered the third approximation, and by Epikhin (1976). These equations, to any order of approximation, were obtained by Pyragas (1973) and Alexandrov and Pyragas (1976). It was shown in those papers that, under the analytical assumption, the equations of the relative motion in some neighborhood of the reference trajectory have the form

\[
\frac{\partial^2 \xi_2}{\partial \xi_1^2} + \tilde{R}_{\gamma\delta\mu\nu} \xi_1^\mu \xi_1^\nu = e F^\gamma_{\xi_1} \left( u^\alpha, \xi_1^\beta, \frac{\partial}{\partial \xi_1^\alpha}, \tilde{R}, \hat{R}; (\ldots) \right),
\]

(2.1)