SUSPENDED PARTICLES AND THE GRAVITATIONAL INSTABILITY OF A ROTATING PLASMA

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Abstract. The gravitational instability of an infinite homogeneous self-gravitating and finitely conducting, rotating gas-particle medium, in the presence of a uniform vertical magnetic field, is studied to include finite Larmor radius and suspended particles effects. The particular cases of the effects of rotation, finite conductivity, finite Larmor radius and suspended particles on the waves propagated along and perpendicular to magnetic field have been discussed. Jeans's criterion determines the gravitational instability.

1. Introduction

Jeans (1902) discovered the gravitational instability of an infinite homogeneous self-gravitating medium. A detailed account of the gravitational instability, under the separate and simultaneous effects of rotation and magnetic field, has been given by Chandrasekhar (1961). It has been found that a uniform magnetic field and rotation (taken together and separately) do not alter Jeans's criterion of instability. The stabilizing effect of finite Larmor radius, which exhibits itself in the form of magnetic viscosity in the fluid equations on plasma instabilities, has been demonstrated by Rosenbluth et al. (1962), Roberts and Taylor (1962), Jukes (1964) and Vandakurov (1964). Sharma (1974) studied the gravitational instability of an infinite homogeneous self-gravitating rotating plasma in the presence of a uniform vertical magnetic field to include finite Larmor radius effects.

Sharma et al. (1976) have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. They have found that the effect of suspended particles is to destabilize the layer and that the magnetic field has a stabilizing influence. In another study Sharma (1975) studied the effect of suspended particles on the gravitational instability of an infinite homogeneous gas-particle medium. In the present paper we study the gravitational instability of a finitely conducting, rotating gas-particle medium in the presence of a uniform vertical magnetic field to include the effects of finite Larmor radius and suspended particles. The uniform magnetic field is taken to be along the z-axis and the axis of rotation is taken along the direction of the magnetic field. The presence of suspended (or dust) particles in the gas in astronomical contexts is more realistic and a reconsideration of the gravitational instability problem by including the effect of suspended particles in the presence of rotation, finite conductivity and finite Larmor radius is certainly called for. This aspect forms the subject matter of the present paper.
2. Perturbation Equations

The linearized perturbation equations for the self-gravitating gas-particle medium in the presence of a uniform vertical magnetic field \( \textbf{H}(0, 0, H) \) and which is rotating about the z-axis with uniform angular velocity \( \Omega(0, 0, \Omega) \) are:

\[
\rho \frac{\partial \textbf{u}}{\partial t} = -\nabla \delta p - \nabla \mathbf{P} + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} + 2\eta (\mathbf{u} \times \Omega) + q\nabla \delta U +
\]

\[
+ q\nu [\nabla^2 \mathbf{u} + \frac{1}{2} \nabla (\nabla \cdot \mathbf{u})] + KN(v - u),
\]

\[
(\tau \frac{\partial}{\partial t} + 1) v = \mathbf{u},
\]

\[
\frac{\partial}{\partial t} \delta \rho = -\rho \nabla \cdot \mathbf{u},
\]

\[
\delta p = c^2 \delta \rho,
\]

\[
\nabla^2 \delta U = -4\pi G \delta \rho,
\]

\[
\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \eta \nabla^2 \mathbf{h},
\]

\[
\nabla \cdot \mathbf{h} = 0;
\]

where \( \delta \rho, \delta p, \delta U, \mathbf{u}(u, v, w) \) and \( \mathbf{h}(h_x, h_y, h_z) \) denote respectively the perturbations in density \( \rho \), pressure \( p \), gravitational potential \( U \), gas velocity and magnetic field \( \mathbf{H} \); \( N(\bar{x}, \bar{t}) \) and \( v(\bar{x}, \bar{t}) \) denote the number density and velocity field of the particles, \( K = 6\pi \eta v \), \( v \) being the particle radius and \( \nu \) the kinematic viscosity of the gas, is a constant and \( \bar{x} = (x, y, z) \), \( c (= \sqrt{\gamma \rho_0 \delta \rho}) \) and \( G \) denote the resistivity, the velocity of sound in the medium and the constant of gravitation respectively. \( mN \) is the mass of particles per unit volume and \( \tau = m/K \).

In writing the linearized perturbation form (2) of the equation of motion for the particles, we have neglected the buoyancy force as its stabilizing effect for the case of two free boundaries is extremely small. Interparticle reactions are also ignored by assuming the distance between particles to be too large compared with their diameter. In writing Equation (1) use has been made of the Stokes’s assumption that the bulk viscosity is zero.

For the vertical magnetic field along the z-axis, the components of pressure tensor \( \mathbf{P} \), taking into account the finite ion-gyration radius, are

\[
P_{xx} = -q\nu_0 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad P_{xy} = P_{yx} = q\nu_0 \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right),
\]

\[
P_{xz} = P_{zx} = -2q\nu_0 \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \right), \quad P_{yz} = q\nu_0 \left( \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \right),
\]

\[
P_{za} = P_{zz} = 2q\nu_0 \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \right), \quad P_{zz} = 0;
\]

where \( q\nu_0 = NT/4\omega_H \), where \( \omega_H \) is ion-gyration frequency, while \( N \) and \( T \) denote the number density and the ion temperature, respectively.