INFECTION OF RELATIVISTIC ELECTRON SPECTRA
FROM MEASUREMENTS OF INVERSE COMPTON RADIATION

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Abstract. The inference of relativistic electron spectra from spectral measurement of inverse Compton radiation is discussed for the case where the background photon spectrum is a Planck function. The problem is formulated in terms of an integral transform that relates the measured spectrum to the unknown electron distribution. A general inversion formula is used to provide a quantitative assessment of the information content of the spectral data. It is shown that the observations must generally be augmented by additional information if anything other than a rudimentary two or three parameter model of the source function is to be derived. It is also pointed out that since a similar equation governs the continuum spectra emitted by a distribution of black-body radiators, the analysis is relevant to the problem of stellar population synthesis from galactic spectra.

1. Introduction

Although much of our astrophysical knowledge derives directly from the analysis of spectral data, in many instances information on source structure is revealed only by the inversion of an integral transform, a process which is beset by many difficulties (e.g., Kunasz et al., 1973; Craig and Brown, 1976). Specifically the inversion lacks stability, so that small changes in the observational data give rise to large changes in the required source function. This is a reflection of the fact that the observations are insensitive to the physics of the source and tends to permit a proliferation, rather than an elimination, of rival theoretical models. Many models can 'fit' the data (see Chandrasekhar and Münch, 1950; Bohm, 1961; Brown, 1975).

In the present paper we consider a typical inverse spectral problem, namely the deduction of relativistic electron spectra from inverse Compton radiation. The photon spectrum is produced by scattering that occurs when relativistic electrons traverse a low temperature black-body radiation field. Although the interpretation of Compton radiation is of interest in both solar and stellar astrophysics (see Korchak and Ponomarenko, 1965) we will concentrate, for definiteness, mainly on the solar application. Under the interpretation of interest, solar flare hard X-ray bursts are attributed to relativistic electrons, accelerated during the initial phases of the flare, interacting with the radiation field of the photosphere (Korchak, 1971; Brown, 1975). Our aim is to explore, within this interpretation, the extent to which Compton radiation can reveal the distribution of relativistic flare electrons.

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However, in formulating this problem our intention is to emphasize, rather than to
elude, the general form underlying all spectral inversions, indeed all 'backward
problems' (see Franklin, 1970; Turchin et al., 1971; Craig, 1977). Accordingly, we
seek to answer three questions, central to any spectral inversion problem:

(i) Under what circumstances is the inversion stable; in other words for what
classes of source function are the data informative?

(ii) How severe is the instability in the practical inversion problem, i.e., how many
independent parameters defining the source function can be derived from data
of a specified accuracy?

(iii) How can the problem be reformulated to make best use of spectral data?

As motivation for the present study we mention that an understanding of what the
data can and cannot establish with respect to the source function is a necessary pre-
requisite for the successful interplay of sound theoretical modelling and constructive
data analysis. This point is further emphasised in the Appendix where we consider a
related problem that arises in the interpretation of Galactic continuum spectra.

2. Background

We follow the general formulation and notation given by Korchak and Ponomarenko
(1965) for the inverse Compton problem. Accordingly non-thermal flare electrons are
presumed to move isotropically in the background radiation field of the solar photo-
sphere. If $E_0$ denotes the energy of a background photon, the radiation field follows
the Planck spectral distribution

$$n = \frac{n_0}{aT^3} \exp\left(\frac{E_0}{T}\right) - 1 \quad (a = 2.404) \quad (2.1)$$

photons per unit $E_0$, where $T (\approx 1$ eV) is the photospheric temperature in energy units
and $n_0$ the total background photon number density in the source. Of prime interest
is the case where relativistic electrons of energy $E \gg mc^2 (\beta = \gamma/c \sim 1)$ interact with
background photons to produce high energy singly scattered photons ($E_0 \rightarrow E_1$)
satisfying the inequalities

$$E_1 \frac{\beta}{T} \gg 1 \quad \text{and} \quad \frac{E_1}{E} \ll 1 + \beta. \quad (2.2)$$

The effective cross-section for production of a photon of energy $E_1$ (after averaging
over the Planck function (2.1)) is then given by

$$\sigma(E_1, E) = \frac{3}{4a} \sigma_T \left(\frac{mc^2}{E}\right)^2 \frac{1}{T} \sum_{k=1}^{\infty} \frac{e^{-kx}}{k^2} g(kx), \quad (2.3)$$