A STUDY OF THE EXPANSION OF THE SOLAR CORONA WITH RADIATION HEAT FLUX

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(Received 7 November, 1979)

Abstract. The expansion of the solar corona, with the aid of hydrodynamic blast wave theory using the concept of the Roche model, is studied here when both the solar gravity and radiation heat flux are taken into consideration.

1. Introduction

Parker (1961) has studied the expansion of the solar corona in the light of blast wave theory. He observed that a sudden rise of temperature to the range of $2-4 \times 10^6$ K should lead to a sudden explosive expansion of the gaseous medium. He therefore applied the hydrodynamic blast wave theory to explain this phenomenon. The model on which he worked is the generalized Roche model, where the mass in the shape of a solid sphere in the central core is constant. The density law of the undisturbed gas is assumed to obey the inverse-square law, distance being measured from the centre of the core. He neglected the effect of solar gravity and also the effect of radiation. After Taylor (1950), similarity transformation was used to reduce the governing equations to ordinary differential equations and they were then solved numerically.

Carrus et al. (1951) have also investigated the Nova phenomenon in the Roche model, but they only considered the effect of gravity due to the core. Deb Ray (1966) gives an analytical solution of such a problem for $\gamma = \frac{4}{3}$ and approximately analytical solutions for $\gamma \neq \frac{4}{3}$.

In this paper we shall consider the same model, but with the effects of solar gravity and radiation heat flux taken into consideration. We have developed similarity solutions when the radiation heat flux is more important than the radiation pressure and radiation energy. The gas in the undisturbed field is assumed to be at rest.

As in previous work, we also have assumed the gas to be grey and opaque, and the shock to be transparent and isothermal. We have also assumed that the total energy of the explosion is constant.

2. Equation of Motion and Boundary Conditions

The equations of flow behind a spherical shock are

$$\frac{d\rho}{dt} + \frac{\rho}{r^2} \frac{\partial}{\partial r} (ur^2) = 0,$$

(1)
\[
\frac{du}{dt} + \frac{1}{\varrho} \frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0, \tag{2}
\]
\[
\frac{dE}{dt} + p \frac{d}{dt} \left( \frac{1}{\varrho} \right) + \frac{1}{\varrho r^2} \frac{\partial}{\partial r} (r^2 F) = 0, \tag{3}
\]

where \(m\) denotes the constant mass of the core; \(u, p\) and \(\varrho\) are the velocity, material pressure and density, respectively, of the gas at radial distance \(r\) from the centre of the core at an instant \(t\), \(G\) being the gravitational constant. \(E\) denotes the material energy per unit mass and \(F\) is the heat flux. Radiation energy and radiation pressure are neglected.

For an ideal gas we have
\[
E = \frac{P}{\varrho (\gamma - 1)}, \quad p = \Gamma \varrho T, \tag{4}
\]

where \(T\) is the temperature, \(\Gamma\) the gas constant and \(\gamma\) the ratio of specific heats.

Assuming local thermodynamic equilibrium, and taking Rosseland’s diffusion approximation, we have
\[
F = -\frac{c \alpha}{3} \frac{\partial}{\partial r} (a_1 T^4), \tag{5}
\]

where \(a_1 c/4\) is the Stefan–Boltzmann constant; \(c\), the velocity of light; and \(\mu\), the mean-free path of radiation, is a function of density and temperature.

Following Wang (1966), we take
\[
\mu = \mu_0 \varrho^{\alpha'} T^{\beta'}, \tag{6}
\]

\(\mu_0, \alpha'\) and \(\beta'\) being constants.

The disturbance is headed by an isothermal shock and the conditions are
\[
\varrho_1 V = \varrho_2 (V - u_2) = m_s, \tag{7}
\]
\[
p_2 - p_1 = m_s u_2, \tag{8}
\]
\[
E_1 + \frac{p_1}{\varrho_2} + \frac{1}{2} V^2 = E_2 + \frac{p_2}{\varrho_2} + \frac{1}{2} (V - u_2)^2 - \frac{F_2}{m_s}, \tag{9}
\]
\[
T_1 = T_2; \tag{10}
\]

where subscripts 1 and 2 are for the regions just outside and just inside the shock surface, respectively, and \(V\) denotes the shock velocity.

In front of the shock, in the undisturbed gaseous medium, we have, by our assumption
\[
\varrho_1 = \frac{\beta}{R^2}, \tag{11}
\]