THE NEED FOR AN ENERGY-DEPENDENT
TORSION-COUPLING CONSTANT IN THE EARLY UNIVERSE

(Letter to the Editor)

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(Received 6 March, 1989)

Abstract. We present arguments to show that torsion-coupling constant (which depends on energy as $E^{-2}$) can pass through the values of the coupling constants of the other interactions during the evolution of the Universe. An energy-dependent torsion-coupling constant helps in a natural way to understand the ratios of the coupling strengths of the different fundamental interactions.

It has been emphasized by many authors (Hehl and Datta, 1971; Sivaram and Sinha, 1975; Kaempffer, 1979; de Sabbata and Gasperini, 1979, 1980) that the general structure of metric-torsion theories allows parity violating interactions, i.e., the torsion of space-time might be responsible for parity violation in weak interactions. It has been recognized that the torsionic contact interaction between two Dirac particles has a formal analogy with weak interaction Lagrangian with the usual Einstein–Cartan static term for the field and can be written as

$$\mathcal{L} = iV_{\mu}(\psi)\dot{J}_{\mu}(\psi^{'}) - A_{\mu}(\psi)J^{5\mu}(\psi^{'}) =$$

$$= (-3/16)\chi[J_{\mu}(\psi)J^{\mu}(\psi^{'}) + J^{5}_{\mu}(\psi)J^{5\mu}(\psi^{'})],$$

$$J^{\mu} = \psi\gamma^{\mu}\psi, \quad J^{5\mu} = \psi\gamma^{5}\gamma^{\mu}\psi, \quad V_{\mu} = Q^{z}_{\mu},$$

$$A_{\mu} = (1/4)\epsilon_{\mu\nu\rho\sigma}Q^{\nu\rho\sigma} \quad (Q^{\nu\rho\sigma} \text{ being the torsion tensor}),$$

$$V_{\mu} = i(3/16)\chi J_{\mu}, \quad A_{\mu} = (3/16)\chi J^{5}_{\mu}. \quad (2)$$

This may be written in the standard $(V - A)$ form if at least one of the two fermions is massless (i.e., described by a two-component spinor $\gamma^{
u\mu}_\lambda \psi = 0$ and $(1 - \gamma^5)\psi = 2\psi$), we have

$$\mathcal{L} = -(3/32)\chi\psi\gamma^\nu\gamma^\mu (1 - \gamma^5)\psi (J^{\mu} + J^{5\mu}) =$$

$$= -(3/32)\chi\psi\gamma^\nu\gamma^\mu (1 - \gamma^5)\psi\gamma^\mu(1 - \gamma^5)\psi. \quad (3)$$

Thus we have a torsionic interaction Lagrangian which is formally identical to the weak interaction four-fermion $(V - A)$ Lagrangian except for the value of the coupling strengths of the different fundamental interactions.

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constant. This raises the possibility that torsion may provide a geometrical model for weak interactions just like curvature does for gravitational forces. However, the coupling constant in the above Lagrangian (3) is

$$\frac{3\chi}{32} = 32\pi K_g h^2/4c^2 \approx 10^{-81} \text{ erg cm}^3,$$

whereas the Fermi weak coupling constant for the four-fermion or \((V-A)\) theory is

$$G_F/\sqrt{2} \approx 10^{-50} \text{ erg cm}^3.$$

Therefore, in order to have a complete identification of torsionic and weak interactions we must postulate a spin-torsion coupling constant which is different from the mass-curvature coupling in Einstein's equations by a factor (Sivaram and Sinha, 1974, 1975; Kaempffer, 1976; de Sabbata and Gasperini, 1978) of

$$\frac{\chi'}{\chi} = \sqrt{\frac{8}{9}} \left( \frac{G_F c^2}{\pi K_g h^2} \right) \approx 10^{31}.\tag{6}$$

One has the possibility of giving arbitrary value to the torsionic coupling constant. There is no compelling \textit{a priori} reason why it should have the same value as the gravitational constant.

In fact one can give some arguments to justify identifying the torsion coupling \(G_T\) to the Fermi \(G_F\). For instance, the similarity to the four-fermionic interaction (i.e., of the torsion interaction of two Dirac particles): i.e., with the action

$$\int d^4x \left( G_T/\sqrt{2} \right) \bar{\psi}_1 \psi_2 \bar{\psi}_3 \psi_4$$

implying that \(G_T\) has the dimensionality of mass to a \textit{negative} power. More precisely: the action is to be dimensionless; \(\psi\) being a spinor has dimensions (mass)\(^{3/2}\) so that \((\bar{\psi}\psi)^4\) has dimensions (mass)\(^6\), \(d^4x\) has dimensions of (mass)\(^{-4}\), so that \(G_T\) must have dimension of (mass)\(^{-2}\).

On the contrary, in the Hilbert Lagrangian for gravity

$$\int d^4x (1/16\pi G) \sqrt{-g} R,$$

the coupling \((1/16\pi G)\) has the dimension of (mass)\(^2\); since \(\sqrt{-g} d^4x\) has dimensions (mass)\(^{-4}\), \(R\) the curvature scalar has dimensions (mass)\(^2\), then for the action to be dimensionless \((1/16\pi G)\) must have dimensions of (mass)\(^2\). This means that the dominant contributions to \(1/16\pi G\) would arise from the higher mass states, the highest masses (i.e., mediating or virtual particles of highest mass) contributing the most. Thus for the Newtonian gravity coupling, the Planck mass states would chiefly determine the value \(1/16\pi G_N\).

\(G_T\) on the other hand in Equation (7) having dimension of (mass)\(^{-2}\), would receive dominant contributions from the lower mass states, i.e., from the intermediate weak bosons \((W, \text{ etc.})\). This would fix its value at the Fermi constant \(G_F\), for intermediate boson masses \(\approx 100 \text{ GeV}\). The whole hierarchy of heavier mass bosons up to Planck