SELF-ORGANIZATION OF ELECTROMAGNETIC WAVES INTO VORTICES IN A MAGNETIZED ELECTRON–POSITRON PLASMA

T. D. KALADZE
1. N. Vekua Institute of Applied Mathematics, Tbilisi State University, Tbilisi, U.S.S.R.

and

P. K. SHUKLA
Ruhr-Universität Bochum, Bochum, F.R.G.

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Abstract. This paper presents a new class of well localized dipolar vortex solutions to the newly derived set of coupled nonlinear equations governing the dynamics of low-frequency electromagnetic waves in a strongly magnetized electron–positron plasma.

Recently, there has been a great deal of interest in investigating dipolar vortices in the planetary atmosphere, rapidly rotating shallow water in laboratory experiments (Nezlin, 1986), as well as in magnetized plasmas (Petviashvili and Pokhotelov, 1986). The nonlinear equations governing the evolution of Rossby waves in the atmosphere and numerous low-frequency waves in magnetized plasmas contain a nonlinearity in the form of two-dimensional vector product (a Jacobian). Larichev and Reznik (1976) pointed out that the latter is very essential for the formation of dipolar vortices. The existence and structure of dipolar vortices in atmospheric and plasma physics have been investigated by a number of authors and the present state of art appears in a topical review article (Petviashvili and Pokhotelov, 1986).

Yu et al. (1986) introduced the notion of dipolar vortices in the study of nonlinear electromagnetic wave phenomena in a strongly magnetized electron–positron plasma. In particular, using the cold plasma and Maxwell equations they derived a pair of coupled nonlinear equations governing the propagation of finite amplitude shear Alfvén-like waves. It has been shown that the nonlinear waves can propagate in the form of two-dimensional dipolar vortices, the outer solution of which has along tail. In this paper we present a new class of dipolar vortex solution to the newly derived set of equations (Yu et al., 1986). Specifically, we construct dipolar vortex solutions which are well localized in the outer region, whereas the inner solutions are different than those of Yu et al. (1986).

We start with the coupled set of nonlinear equations (Yu et al., 1986)

\[ a \frac{d}{dt} \nabla^2 \phi = -c \frac{d}{dz} J, \quad (1) \]

\[ \partial_z A - \lambda^2 \frac{d}{dz} J = -c \frac{d}{dz} \phi, \quad (2) \]
which govern the dynamics of shear Alfvén-like electromagnetic waves in a strongly magnetized electron–positron plasma. Here, \( a = 1 + \omega_p^2/\Omega^2 \), \( \omega_p(\Omega) \) is the magnetic field \( B_0(\Omega) \) component of the vector potential, and \( \phi \) is the scalar potential. We have defined \( d_i = \partial_i + cB_0^{-1}\partial_x \times \nabla\phi \cdot \nabla \) and \( d_z = \partial_z - B_0^{-1}\partial_x \times \nabla A \cdot \nabla \).

We look for stationary two-dimensional localized solutions of (1) and (2) by assuming that the variables \( \phi \) and \( A \) are functions of \( x \) and \( \xi = y - ut + \alpha z \). Then, the solutions of (1) and (2) can be put in the form

\[
\nabla^2 \phi = (b_\phi \tilde{\phi} + b_e \tilde{A})/a\lambda^2 ,
\]

(3)

\[
J = (\tilde{A} - b_e \tilde{\phi}) ,
\]

(4)

where \( \tilde{\phi} = \phi - B_0ux/c, \tilde{A} = A - B_0xx, \) and \( u \) and \( \alpha \) are constants.

The dipolar vortex solutions of (3) and (4) can be constructed following standard methods (Larichev and Reznik, 1976; Petviashvili and Pokhotelov, 1986). Accordingly, we introduce the polar coordinates \( r = (x^2 + \xi^2)^{1/2} \) and \( \theta = \arctan(\xi/x) \) and divide \( (r, \theta) \)-plane into two parts: an outer region \( r > R \) and an inner region \( r < R \), where \( R \) corresponds to the vortex radius. In the outer region, we assume \( \phi = A = 0 \), so that from (3) and (4) we find that

\[
b_e^2 = \omega c/u \quad \text{and} \quad b_\phi^2 = -\alpha^2 c^2/u^2 ,
\]

(5)

and the outer solutions can be written as

\[
\phi = e_1 K_1(sr) \cos \theta , \quad A = a_1 K_1(sr) \cos \theta ;
\]

(6)

where \( e_1 \) and \( a_1 \) are constants and \( K_1 \) is the McDonald function. Inserting (6) into (3) and (4), using (5), we obtain an algebraic system for determining the coefficients \( e_1 \) and \( a_1 \). The condition of its solvability gives

\[
s^2 = (1 - \alpha^2 c^2/au^2)/\lambda^2 .
\]

(7)

The outer solutions are localized provided that \( s^2 > 0 \), yielding \( u^2 > \alpha^2 c^2/a \). Note that the present solution (6) does not have a long tail in contrast to Yu et al. (1986).

In the inner region, the value of \( b_e^2 \) given in (5) remains the same but the value of \( b_\phi \) is different. The solutions of (3) and (4) in the inner region can be written as

\[
\tilde{\phi} = [e_2 J_1(k_1 r) + e_3 J_1(k_2 r)] \cos \theta ,
\]

(8)

\[
\tilde{A} = [a_2 J_1(k_1 r) + a_3 J_1(k_2 r)] \cos \theta ;
\]

(9)

where \( e_2, e_3, a_2, a_3 \) are constants, \( J_1 \) is the Bessel function of the first order, while \( k_1 \) and \( k_2 \) represent inner wave numbers. Inserting (8) and (9) into (3) and (4), we find a system of equations

\[
e_2(k_1^2 + b_\phi/a\lambda^2) + a_2 b_e^2/a\lambda^2 = 0 ,
\]

(10)

\[
-e_2(b_e^2/\lambda^2) + a_2(k_1^2 + 1/\lambda^2) = 0 ,
\]

(11)