Abstract. Weight functions for the determination of the periods of linear adiabatic non-radial oscillations have been calculated in the same manner as Epstein's classic treatment of purely radial oscillations. Quadrupole ($l=2$) oscillations for the $f$ and lower order $p$ and $g$-modes were considered. One group of static models were polytropes in the range $1.0 < n < 4.0$ with $\Gamma_1 = \frac{5}{3}$; thus included were configurations that were convectively stable, unstable and neutrally stable throughout. Another group consisted of $n = 3.0$ polytropes with convective shells or convective cores; $\Gamma_1$ was set at different values in each region in order to produce stability ($\Gamma_1 = \frac{5}{3}$) or instability ($\Gamma_1 \leq \frac{5}{3}$). The weight function provides a pictorial means for assessing the relative importance of each region of a given static model with respect to generating a given non-radial mode.

1. Introduction

Epstein (1950) used the variational principle associated with the linear adiabatic radial wave equation to compute 'weight functions' for the determination of the periods of these oscillations. For a particular mode and equilibrium model this function revealed, with respect to radial distance, which portions of the star contributed most to the formation of the period. These weight functions are of value in constructing theoretical models for variable stars, since these functions reveal which portions of the star are important and must be treated carefully in the static model, and which are unimportant and thus can be roughly approximated.

As with the radial case, the wave equation for linear adiabatic non-radial oscillations can also be written as an eigenvalue equation with a Hermitian operator, and has an associated variational principle (Chandrasekhar, 1964). The variational formulation of the eigenvalue problem, given in Section 2, involves the simultaneous variation of two functionals, with the eigenvalue equal to the ratio of the two. The integrand of the numerator is the weight function for the period, and integrand of the denominator is a normalization function. In this paper, these two functions are calculated for the $f$-mode and low order $p$ and $g$-modes of quadrupole ($l=2$) oscillations. One group of static models are polytropes of index $n$, in the range $1.0 \leq n \leq 4.0$, where the adiabatic exponent $\Gamma_1$ is fixed at $\frac{5}{3}$. These models include those that are convectively stable, unstable, and of neutral stability. Results for these models appear in Section 3a. Another group consists of polytropes with convective regions. Goossens and Smeyers (1974, hereafter referred to as GS) did a similar investigation pertaining to gravity
modes in polytropes with convective shells. Their static models had a fixed value of \( \Gamma_1 \) throughout, with \( n \) chosen in each region to give stability or instability. Here we take the opposite approach: \( n \) is kept fixed, and \( \Gamma_1 \) takes on different values in each region. Section 3b contains the results for this group.

2. Basic Equations and Methods of Computation

The equations governing polytropes are well known and will not be repeated here (see, e.g. Cox and Giuli, 1968, Section 23.1). For polytropes, the Schwarzschild criterion for convective stability, \( A < 0 \), where

\[
A = \frac{1}{\rho} \frac{d\rho}{dr} - \frac{1}{\Gamma_1 \rho} \frac{dp}{dr},
\]

reduces to

\[
\Gamma_1 > \frac{n+1}{n},
\]

where \( n \) is the polytropic index and

\[
\Gamma_1 = \left( \frac{\partial \ln \rho}{\partial \ln \rho} \right)_s.
\]

The first group of static models were polytropes in the range 1.0 \( \leq n \leq 4.0 \) with \( \Gamma_1 = \frac{3}{2} \) throughout. Thus included were stable \( (n > \frac{3}{2}) \), neutrally stable \( (n = \frac{3}{2}) \), and unstable \( (n < \frac{3}{2}) \) configurations. The second group consisted of \( n = 3.0 \) polytropes with \( \Gamma_1 \) fixed at different values in each region to produce stability \( (\Gamma_1 = \frac{3}{2}) \), neutral stability \( (\Gamma_1 = \frac{3}{2}) \), or instability \( (\Gamma_1 < \frac{3}{2}) \). This approach is realistic if one wishes to simulate a convective shell produced by a hydrogen or helium ionization zone. The shell would thus have to be located near the surface of the static model. Here, however, we simply acknowledge the \textit{ad hoc} nature of this technique and use it for any type of convective region, including a convective core. Note that, since \( n \) is fixed at the same value throughout the model, there is no need to match polytropic variables at the radiative zone/convective zone interfaces, as is required by the method of GS; construction of the static model is thus much simpler by this method.

The equations governing linear adiabatic non-radial oscillations of a gaseous star have been discussed by Ledoux and Walraven (1958). The form of the equations and the method of computation employed here are exactly those of Osaki and Hansen (1973). A few of the more important aspects of their treatment will be reiterated. The fourth order system is put in the form of four first order differential equations in the four variables

\[
y_1 = \frac{c_r}{r}, \quad y_2 = \frac{1}{gr} \left( \frac{p'}{\rho} + \phi' \right),
\]

\[
y_3 = \frac{1}{gr} \phi', \quad y_4 = \frac{1}{g} \frac{d\phi'}{dr},
\]

(4)