II. Study of Different Velocity Fields and Luminosity Flux Function

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Abstract. Considering the azimuthal velocity fields for different radial dependence we obtain the pressure profiles for the thin disk using the general formalism obtained earlier and further look at the profiles of the luminosity flux function using the approach as given recently by Hanawa (1988). It appears that the profile of this function is not very sensitive to change in the \( r \)-dependence of the velocity fields.

1. Introduction

In an attempt to understand the dynamics of accretion disks containing plasma we have recently given a general formalism, in the framework of general relativity, using the linearised Kerr-background, of the description of a magnetofluid with self-consistent electromagnetic fields (Prasanna and Bhaskaran, 1989; Paper I). In the case of infinite conductivity we had seen that for a suitably modified dipole type of magnetic field and the corresponding electric field, obtained through force-free condition, one could have a class of azimuthal velocity fields with one of them being Keplerian, when both the radial and the meridional velocities are considered negligible.

In what follows we shall consider a thin disk of incompressible fluid \( (\rho = \text{const.}) \) and obtain the pressure profiles for different values of \( n \) and \( \beta \) which control the radial dependence of \( V(\phi) \) and the angular momentum of the fluid element. Furthermore, we also consider the nature of luminosity function associated with the radiative flux adopting the method due to Hanawa (1988), for different values of \( V(\phi) \).

2. Disk Structure

The general equations developed in Paper I admit, for a dipole type of magnetic field as given by

\[
B(r) = \frac{B_0 R^3}{r^3} \cos \theta, \\
B(\theta) = \frac{B_0 R^3}{2r^3} \left( 1 - \frac{2m}{\gamma} \right)^{1/2} \sin \theta, \\
B(\phi) = 0,
\]

(2.1)

a class of velocity fields

\[
V(\rho) = 0, \quad V(\theta) = 0, \\
V(\phi) = K (1 - 2m/r)^{-1/2} r(3-n)/2 \sin^n \theta
\]

(2.2)
for a current distribution \( J^{(\phi)} = (0, 0, J^{(\theta)}, J^{(r)}) \)

\[
J^{(\phi)} = -\frac{3mB_0 R^3}{r^5} \sin \theta,
\]

\[
J^{(r)} = -\frac{KB_0 R^3}{r^{(n+3)/2}} (1 - 2m/r)^{-1/2} \sin^n \theta \times
\]

\[
\times \left\{ \left( \frac{1 - n}{4} \right) (1 - 2m/r) + 1 \right\} \sin^2 \theta - (n + 1) \cos^2 \theta \},
\]

(2.3)

and the electric field \( E \)

\[
E_{(\phi)} = 0,
\]

\[
E_{(r)} = \frac{KB_0 R^3}{2r^{(n+3)/2}} \sin^{(n+1)} \theta,
\]

(2.4)

\[
E_{(\theta)} = -\frac{KB_0 R^3}{r^{(n+3)/2}} (1 - 2m/r)^{-1/2} \sin^n \theta \cos \theta.
\]

The associated momentum equations are then given by

\[
(\rho + P/c^2) \left( 1 - \frac{V^{(\phi)^2}}{c^2} \right)^{-1} \left[ \frac{mc^2}{r^2} - (1 - 2m/r) \frac{V^{(\phi)^2}}{r} \right] +
\]

\[
+ (1 - 2m/r) \frac{\partial P}{\partial r} = \frac{1}{c} (1 - 2m/r)^{1/2} [B_{(\theta)} J^{(\phi)} - E_{(r)} J^{(\theta)}]
\]

(2.5)

and

\[
(\rho + P/c^2) \left( 1 - \frac{V^{(\phi)^2}}{c^2} \right)^{-1} \frac{V^{(\phi)^2}}{r} \cot \theta - \frac{1}{r} \frac{\partial P}{\partial \theta} =
\]

\[
= \frac{1}{c} [B_{(\phi)} J^{(\phi)} + E_{(\theta)} J^{(\theta)}],
\]

(2.6)

which for thin disk approximation \( (\theta = \pi/2) \) reduce to

\[
\frac{\partial P}{\partial \theta} = 0,
\]

(2.7)

\[
\frac{dP}{dr} + (1 - 2m/r - \frac{K^2 r^{(3-n)}}{c^2} \left( \frac{mc^2}{r^2} - \frac{K^2 r^{3-n}}{r} \right)) (\rho c^2 + P) =
\]

\[
= \frac{B_0^2 R^6}{2cr^n} \left[ \frac{K^2}{r^{n-3}} \left\{ \left( \frac{1 - n}{4} \right) + (1 - 2m/r)^{-1} \right\} - 3m/r \right].
\]

(2.8)