NON-RADIAL OSCILLATIONS AND STABILITY OF PRENDERGAST’S MODEL

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Abstract. The frequencies of the linear and adiabatic oscillations of Prendergast’s model are determined with the aid of a perturbation method. The influence of the magnetic field on the frequencies of the different types of spheroidal oscillation modes is discussed.

1. Introduction

In two previous papers (Goossens, 1972; Goossens et al., 1976; hereafter referred to as Paper I and Paper II, respectively) a perturbation method has been developed which allows the determination of the influence of a weak magnetic field on the eigenfrequencies of the linear and adiabatic oscillations of a gaseous star, exact to the first order in \( M/|W| \) (\( M \) representing the magnetic energy and \( W \) the gravitational potential energy). Moreover, the perturbation method has been used for the computation of the oscillation frequencies of Ferraro’s model (Ferraro, 1954). This model is an axisymmetric homogeneous oblate spheroid. Inside the material configuration a purely poloidal magnetic field prevails due to an azimuthal electric current distribution that is proportional to the distance from the axis of symmetry. At the spheroidal boundary the internal field links up continuously with an external curl free dipole-type field.

Our aim in the present paper is to determine the oscillation frequencies of Prendergast’s model (Prendergast, 1956, 1958) with the aid of the perturbation method. Prendergast’s model consists of a homogeneous axisymmetric sphere pervaded by a magnetic field with both a poloidal and a toroidal component. The magnetic field vanishes on the spherical boundary and in the external vacuum. In the next Section we briefly review the structure of Prendergast’s model. In Section 3 we apply the perturbation method. Finally, in Section 4 we present our results and we discuss the degree of accuracy of the frequencies obtained by other methods.

2. The Equilibrium Configuration

Prendergast (1956, 1958) studied an axisymmetric and spherical configuration of uniform density. The magnetic field is composed both of a poloidal and a toroidal component

\[
H = KR^2 \left[ \frac{2\gamma(x)}{x^2} \mu \mathbf{l}_r - \frac{1}{x} \frac{d\gamma}{dx} (1 - \mu^2)^{1/2} \mathbf{l}_\theta + \beta \frac{\gamma(x)}{x} (1 - \mu^2)^{1/2} \mathbf{l}_\phi \right].
\]

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$R$ is the radius of the configuration, $x = r/R$ and $\mu = \cos \theta$. $\gamma(x)$ is a function of $x$ alone and is defined by

$$\gamma(x) = \frac{x^2}{\beta^2} \left[ 1 - x^{-3/2} \frac{J_{3/2}(\beta x)}{J_{3/2}(\beta)} \right],$$

where $J_{3/2}$ is the Bessel function of order 3/2. The constant $K$ determines the strength of the impressed magnetic field, while the parameter $\beta$ characterizes the structure of the field and is a root of the equation

$$J_{5/2}(\beta) = 0,$$

where $J_{5/2}$ is the Bessel function of order 5/2. From (2) and (3) it can readily be verified that the radial function $\gamma(x)$ and its first derivative vanish on the free surface $r = R$. In vacuo the external magnetic field is identically zero, so that the magnetic field (Equation (1)) lies wholly within the gaseous sphere.

The behaviour of the gravitational potential inside the configuration corresponds simply to that of a spherically symmetric homogeneous configuration: namely,

$$\Phi(x) = 2\pi G \rho R^2 \left( \frac{3}{4} x^2 - 1 \right).$$

The pressure distribution is given by

$$p(x, \mu) = \frac{3}{2} \pi G \rho^2 R^2 \left\{ (1 - x^2) - h \frac{2\beta^2}{3} \gamma(x) [1 - P_2(\mu)] \right\},$$

where $G$ is the universal constant of gravitation and $h$ represents the ratio of the magnetic energy $M$ to the gravitational potential energy $|W|$ as given by

$$h = \frac{M}{|W|} = \frac{3K^2 R^2}{8\beta^2 \pi^2 G \rho^2}.$$  

3. Oscillation Frequencies of Prendergast’s Model

We want to determine the oscillation frequencies of Prendergast’s model by using the perturbation method we have previously developed. The application of this method to Prendergast’s model is greatly simplified by the characteristic features of the equilibrium configuration. The most essential simplification results from the spherical form of the boundary surface. Because of this property, we can set the function $a(\theta)$ equal to zero in Equation (10) of Paper I and we can use a reference sphere (without any magnetic field) which has the same radius as Prendergast’s model. Secondly, the mass density being constant in Prendergast’s model, the behaviour of the gravitational potential agrees with that in a spherically symmetric homogeneous sphere. When we choose a reference sphere which has, in addition, the same mass density as Prendergast’s model, the only volume perturbations to be considered are those due to the local change in pressure and to the Lorentz force. Finally, we note that the magnetic field vanishes at the boundary surface, so that there is no change in the condition at the sur-