SOME SIMPLE LIMITS ON SIZE AND ROTATION SPEED OF ASTROPHYSICAL OBJECTS EMITTING BY ACCRETION

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Abstract. We demonstrate that the so-called 'force balance point', due to either magnetic fields (then known more specifically as the Alfvén point) or to co-rotation (Roche point), together with the concepts of flux and mass conservation, allows one to place interesting limits on both the size and angular velocity of astrophysical objects emitting by mass accretion. We give three numerical examples, which would seem to be of some astrophysical interest.

1 Introduction

With the discovery of binary variable X-ray sources astrophysical attention concerning the possibility of material accretion as the driving mechanism for the high luminosity outputs (of order $10^{38}$ erg s$^{-1}$), has come to the fore in the last few years. The purpose of the present paper is to show that by appealing to a 'braking' mechanism for the infalling ionized matter, either through the agency of a magnetic field embedded in the underlying star or through angular momentum considerations, some interesting limits can be placed on the size and rotation speed of the underlying star. Further, if the object possesses a strong magnetic field, and if the surrounding ionized 'braked' material co-rotates, then the spatial distribution of the material has an unusual shape under particular conditions on the magnetic field structure of the star, as we shall also presently demonstrate.

We do not know too much about the precise values of the radius, mass and magnetic field strength for the underlying star. For instance in the case of pulsars, magnetic dipole field strengths in the range $10^{10}$-$10^{14}$ gauss are not uncommonly suggested. It would, then, seem that there will be some considerable 'play' in any results we derive for size and angular velocity. However, the dependence turns out not to be too sensitive to, say, the luminosity and magnetic flux as we shall also demonstrate.

Consider, then, material falling onto a star, of radius $R$, magnetic field strength $B$, mass $M$. At a radius $r$ from the star let the material have density $\rho(r)$, velocity $v(r)$. The final temperature attained by a particle of mass $m$ in the material when it eventually hits the surface of the star is

$$kT = GMm/Rs = \frac{1}{2}mv(r)^2. \tag{1}$$

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The mass infall rate is about
\[ \dot{M} = 4\pi R^2 q V; \]
the conserved magnetic flux is about
\[ \Phi = B_s R_s^2 = B(\nu)r^2; \]
and the luminosity of the object is then about
\[ L = v(r)^2 \dot{M}(r/r_s). \]
Armed with these simple quantities we can now ask that the infalling material be
'braked' in one of two ways, magnetic braking or collisions followed by centrifugal
balance.

2. Size and Spin Estimates

A. MAGNETIC BRAKING

In this case there must exist a radius \( r_0 \) (the Alfvén radius), at which
\[ \frac{1}{2} q v^2 = B^2 / 8\pi. \]
Using (1), (2), and (3) in (4) we can write
\[ \frac{r_0}{R_s} = \left( \frac{R_c}{R_s} \right)^{5/3}, \]
where
\[ R_c = \Phi^{4/5} L^{-2/5}(GM)^{1/5}. \]
Clearly, if the bulk velocity of the material is to be braked then \( r_0 \) must exceed \( R_s \),
the radius of the star. Then from (5) we have
\[ R_c \gtrsim R_s, \]
i.e.
\[ R_s \lesssim \Phi^{4/5} L^{-2/5}(GM)^{1/5} \equiv R_{s0}. \]

B. CENTRIFUGAL BRAKING

In this case one argues that there are sufficient plasma-wave ‘collisions’ or individual
particle collisions that the incident material eventually accretes in a co-rotating ring in,
say, the spin equatorial plane of the star.
If so, then for each particle we have
\[ \frac{v_0^2}{r_1} = \frac{GM}{r_1^2} \]
* In view of our ignorance concerning the numerical values pertaining to such stars we do not
believe it would be profitable to deviate from the simplest geometry we can think of. Accordingly we
have chosen spherically symmetric infall as illustrative and possibly typical.