HEATING OF THE SOLAR CHROMOSPHERE AND CORONA

I. Generalized Inhomogeneous Wave Equation for Magnetoacoustic Motions

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(Received 14 October, 1975)

Abstract. The generalized inhomogeneous wave equation that governs magnetoacoustic, vortical and thermal motions in compressible fluids and that thus is applicable to the problem of the heating of the solar chromosphere and corona is obtained. The effects of kinematic and bulk viscosity, heat conduction, Joule dissipation and magnetic diffusivity are included. Under the usual assumptions, the generalized wave equation reduces to the well-known equations of Lighthill, Kulsrud, Phillips and others. The major problems encountered in applying Lighthill's mechanism to sound generation in turbulent media are reviewed for both the subsonic and supersonic cases.

1. Introduction

One of the fundamental problems in solar physics is the heating of the solar chromosphere and corona. This problem was recently reviewed by Leibacher (1973) and Souffrin (1973), who summarized the different physical processes that are important in the generation, propagation and dissipation of waves in the solar atmosphere. One of the most important mechanisms for heating is through the dissipation of sound (acoustic) waves that are generated by the velocity fluctuations of solar granules.

This mechanism was suggested by Biermann (1946, 1948), and also independently by Schwarzschild (1948), and the complete mathematical theory was formulated by Lighthill (1952). Lighthill developed his theory in order to understand aerodynamic noise generated by the motion of a jet in the air.

Proudman (1952) investigated the sound generated by the isotropic turbulent motion of eddies in a uniform acoustic medium, and obtained a relation for the total energy generated in per unit volume of the form

\[ P = \mathcal{L} \frac{\rho_0 \langle v^2 \rangle^{3/2}}{Lc}, \]

where \( \rho_0 \), \( \langle v^2 \rangle^{1/2} \), \( L \) and \( c \) are the density, root-mean-square velocity, characteristic length of the eddies, and the velocity of sound, respectively. Furthermore, \( \mathcal{L} \) is Lighthill's constant which depends on the turbulence model. For Heisenberg's spectrum, \( \mathcal{L}_1 = 37.5 \), and for the isotropic and spherically symmetric Gaussian distribution of turbulent eddies, \( \mathcal{L}_2 = 13.5 \). It is important to mention that this power formula holds only for subsonic motions of turbulent eddies. We can integrate the last

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power relation of the mechanical energy flux (ergs cm\(^{-2}\) sec\(^{-1}\)), and obtain the expression that is commonly used in astrophysics to describe the mechanical energy flux of the solar chromosphere and corona.

Kulsrud (1955) generalized Proudman's work by including the effects of magnetic field. He showed that if the magnetic energy is comparable to the turbulent kinetic energy, the magnetic turbulent motion of eddies dominates the generation of sound waves. In this case Lighthill's constant \(L_2 = 136.8\), which is about ten times larger than Proudman's value. Ffowcs Williams (1963) and Ffowcs Williams and Hawkins (1968) discussed the noise from turbulence convected at high speeds. In this important investigation Ffowcs Williams (1963) corrected an error in Lighthill's original work regarding the Doppler effects encountered between the motion of jets and the convective motion of eddies. Recently, Goldstein (1974) discussed the effects of anisotropic turbulent motion of eddies on aerodynamic noise (Lighthill, 1952; Ribner, 1969), which is based on the earlier work of Batchelor (1946). The directional dependence of the emission of acoustic noise (anisotropic turbulence) was applied to the solar heating problem by Kuperus (1972). However, this problem has not as yet been considered quantitatively in solar physics.

So far all papers mentioned above are valid only for the subsonic motions of the turbulent eddies in a uniform acoustic medium. The fundamental investigation on the generation of sound waves in the supersonic shear layers in non-uniform media was made by Phillips (1960). He obtained an asymptotic solution of the wave equation (Equation (26) in the text) for large values of the Mach number \(M\). So far no exact solution of this wave equation is known. Recently Pao (1971, 1972) developed a generalized theory of aerodynamic noise based on the work of Phillip's, but still the exact acoustic energy formula for the transonic \((M = 1)\) and supersonic motions \((M > 1)\) of the turbulent eddies is not known. There are also several very good review articles on this subject by Lighthill (1962, 1963), Ribner (1964), Ffowcs Williams (1969), Doak (1972), Fuchs and Michalke (1973), and Goldstein (1974).

The first serious attempts at applying Lighthill's mechanism to the heating of solar chromosphere and corona were made by Osterbrock (1961) and also independently by de Jager and Kuperus (1961). Unno (1964) showed that the effect of gravity increases the dipole emission of acoustic noise. The inclusion of gravity was also further investigated more quantitatively by Stein (1967). The application of Lighthill's mechanism was also discussed by Lighthill himself rather pessimistically in 1967. Lighthill's primary criticisms centered on the insufficient generation of acoustic energy by isotropic turbulent motions of granules in the solar convection zone. His mechanism provides insufficient heating of the solar chromosphere and corona. He stressed the importance of the generation of gravity waves above the turbulent convection zone. Theories involving the mechanical heating of the chromosphere and corona by wave motions have been reviewed by Jordan (1968), Kuperus (1969), Stein and Leibacher (1974), Leibacher (1973), Souffrin (1973), and Michalitsanos (1973).

In this paper we investigate the problems encountered with Lighthill's mechanism,