CURRENT GRADIENT-DRIVEN LINEAR AND NONLINEAR ELECTROMAGNETIC WAVES IN A MAGNETIZED ELECTRON–POSITRON PLASMA

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Abstract. A set of coupled nonlinear differential equations which govern the dynamics of finite amplitude electromagnetic waves in the presence of an external current gradient in a magnetized electron–positron plasma has been derived. It is shown that the current gradient can make shear Alfvén-like waves unstable. A quasi-stationary solution of the mode-coupling equations is the well-localized dipole vortex. Application of our results to plasma transport in the pulsar magnetosphere is briefly discussed.

1. Introduction

It is widely thought that the polar cusp region of the pulsar magnetosphere contains a strongly magnetized electron-positron plasma. The latter is created by the cascade breeding of electron-positron pairs along the magnetic field lines (Sturrock, 1971).

An electron–positron plasma supports a great variety of electrostatic as well as electromagnetic oscillations whose characteristics are summarized in several review articles (e.g., Lominadze et al., 1983; Shukla et al., 1986a). It appears that the linear properties of the normal modes in an electron–positron plasma are significantly different from those of the electron–ion plasma, although the underlying physics of the modes and their instabilities are similar.

Recently, attempts have been made in understanding the phenomena of pulsar radiation in terms of nonlinear effects. A number of authors (Yu et al., 1984; Mikhailovskii et al., 1985a; Stenflo et al., 1985; Shukla et al., 1986a) have investigated the modulation of large amplitude electromagnetic waves in a strongly magnetized electron–positron plasma. It has been shown that this interaction may lead to sharply localized wave packets (envelope solitons) which could be associated with the observed pulsar radiation. However, these studies are restricted to wave propagation either parallel or perpendicular to the external magnetic field.

In reality, multi-dimensional perturbations can exist. Yu et al. (1986) considered two-dimensional effects on the nonlinear electromagnetic wave propagation in a strongly...
magnetized electron–positron plasma. Using the hydrodynamic and the Maxwell equations, they have derived the mode coupling equations for an obliquely propagating shear Alfvén-like wave. The dispersion relation of the latter differs considerably from that of the electron–ion plasma. Furthermore, it has been demonstrated that a stationary solution of the mode coupling equations is the two-dimensional dipole vortex which is not well localized. Yu et al. (1986) speculated that such dipolar vortices can affect the transport properties of the pulsar magnetosphere.

In view of the increasing interest in the nonlinear wave motion in an electron–positron plasma, we investigate the linear as well as the nonlinear properties of the electromagnetic radiation in the presence of free-energy sources. We show that the presence of a current gradient may cause the modified shear Alfvén wave to become unstable. Furthermore, it is shown that the current gradient can lead to solitary electromagnetic vortices which are well localized.

The paper is organized in the following fashion. In the next section we derive the mode coupling equations including an external current gradient. Section 3 contains the linear dispersion relation for the electromagnetic waves and the conditions under which they can become unstable in a strongly magnetized electron–positron plasma. The quasi-stationary structures of the nonlinear mode coupling equations are analyzed in Section 4. Here we show the existence of two-dimensional dipolar vortices which could be the consequence of the unstable electromagnetic waves. These solutions, which are well localized, differ from those (Yu et al., 1986) of the shear Alfvén vortices in the absence of the current gradient. Applications of our results to particle transport in the pulsar magnetosphere is briefly discussed in the last section.

2. Basic Equations

We consider a cold electron–positron plasma embedded in an external magnetic field \( B_0 = \hat{z}B_0 \). A background current \( J_0 = \hat{z}J_0(x) \) is aligned along \( B_0 \). In our analysis, we shall assume a linear gradient to exist and use a local approximation around \( x = 0 \). Thus, one can neglect the equilibrium magnetic field \((\propto x^2)\) associated with \( J_0 \). The dynamics of low (compared to the gyrofrequency \( \Omega = eB_0/mc \)) frequency wave motion is governed by the continuity equation

\[
\partial_t n_j + \nabla \cdot (n_j v_j) + \partial_z (n_j v_{jz}) = 0 ,
\]

the parallel component of the momentum equation

\[
(\partial_t + v_{j\perp} \cdot \nabla + v_{je} \partial_z) v_{jz} = \frac{q_j}{m_j} \left( E_z + \frac{1}{c} v_{j\perp} \times B_{\perp} \cdot \hat{z} \right) ,
\]

and Poisson’s equation

\[
\nabla \cdot E = 4\pi e (n_e - n_p) ,
\]

where \( j = e, p \) for electrons and positrons, \( \nabla_{\perp} = \hat{x} \partial_x + \hat{y} \partial_y \), \( E_z \) is the parallel