RELATIVE CHAOS IN STELLAR SYSTEMS

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Abstract. Statistical properties of many-dimensional dynamical systems—stellar systems of different types, are investigated by means of new definition of relative chaos based on the estimation of the Ricci curvature in the direction of the velocity of geodesics. Numerical experiment is performed to calculate the Ricci and scalar curvatures for systems with equal total energy. The results of calculations enable one to obtain schematic classification of stellar systems by increasing degree of chaos.

1. Introduction

The development of computer techniques makes the investigation of complex dynamical systems by means of numerical methods possible. This direction turned out to be quite fruitful and already led to a number of interesting and even unexpected results. Already in a pioneer study of Fermi et al. (1955) unexpectedly slow mixing was discovered in a one-dimensional system of nonlinearly interacting oscillators.

An important field of application of numerical analysis is the N-body problem—particularly the problem of evolution of stellar systems, clusters, and galaxies: stellar systems, being classical examples of collisionless systems of N-gravitating bodies regardless of their apparent simplicity, are considered one of most difficult for investigation of objects in the Universe (Contopoulos, 1984; Gurzadyan, 1987).

The application of numerical methods to stellar systems was at first restricted mainly to following in time the variation of the shape and other observable characteristics of the given system, or of several systems (see, e.g., Eneev et al., 1973); the main difficulty being the necessity of integration of too many equations of motion.

Simultaneously, another direction of numerical investigations began to develop which included the understanding of qualitative-statistical and other properties of Hamiltonian dynamical systems—in particular of stellar systems.

The main feature of this direction is the use of new methods of numerical analysis including criteria of stochasticity and regularity as well as ways of representation of the final information. Thus, in their well-known paper, Hénon and Heiles (1964) used the Poincaré section method for the analysis of two-dimensional system with cubic potential (models of spiral galaxies). Chirikov (1979), Zaslavsky (1984), Benettin et al. (1978), Contopoulos et al. (1978), developed and applied for specific systems strong methods based on the calculation of Lyapunov characteristic numbers and Krylov–Kolmogorov–Sinai (KS) entropy.

In the present paper we use a method of investigation of statistical properties of many-dimensional systems based on the calculation of the so-called Ricci curvature. The idea of the method is that as is well known, motion equations of a Hamiltonian
system can be reduced to equations of geodesics of certain Riemannian manifold. As described in Section 2, the value of the Ricci curvature of this manifold can give a definite information on statistical properties of the Hamiltonian system. This paper is the development of papers (Gurzadyan and Savvidy, 1984, 1986; Gurzadyan and Kocharyan, 1986a), where the stellar systems as geodesic flows were investigated; in Gurzadyan and Savvidy (1986) it was also proposed a method of numerical analysis using two-dimensional curvature.

The problem formulated and investigated in the paper by the method mentioned is as follows: to determine the relative degree of instability (chaos) of several many-dimensional dynamical systems, namely stellar systems with different spatial structure, distribution of stellar velocity vectors, angular moment, etc. The main purpose here consists in a possibility to get a sort of quantitative estimation of degree of chaos of N-dimensional system. Definition of 'relative chaos' given in Section 2 and the corresponding quantitative estimate are based just on the calculations of the Ricci curvature.

The reason of resorting to a numerical method not used earlier is the fact that, in general, the methods mentioned above are not effective for many-dimensional systems. Thus, in calculations of Lyapunov numbers, difficulties can appear not only in connection with proof of the existence of corresponding limits, but also with the interpretation of those numbers themselves.

Results of computer experiments enabled us to get schematical classification of stellar systems of different types by degree of instability. In particular, it turned out that the existence of a central mass in the systems leads to a sufficient instability; and it is remarkable that this conclusion does not depend on system's energy, angular moment, stellar velocity distribution, etc. Analogously, disk systems turned out to be less chaotic than spherically-symmetrical ones; etc.

In total, the results of analysis demonstrate that calculation of the Ricci curvature in the direction of the geodesic's velocity can be an effective method for investigation of chaos in many-dimensional dynamical systems.

The outline of the paper is as follows. In Section 2 the formalism of the Ricci curvature is developed, the definition of relative chaos (instability) is given together with necessary formulae. Results of numerical experiments are presented in Section 3 and are analyzed in Section 4. Discussion of the main conclusions both from the methodical and physical viewpoint is carried out in Section 5. Some alternative definitions of chaos and relations between them are discussed in Appendix 1. In Appendix 2 we discuss the meaning of given definition of chaos for simple systems.

2. The Ricci Curvature

Consider the N-body system with Hamiltonian

$$H(p, q) = \sum_{k=1}^{3N} \frac{p_k^2}{2} + V(q),$$

$$p = (p_1, \ldots, p_k), \quad q = (q_{3N}, \ldots, q_{3N});$$

(2.1)