A LOWER LIMIT TO THE MASS OF NEUTRAL LEPTONS AS CONSTITUENTS OF THE DARK HALO AROUND GALAXIES

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Abstract. We have found that the structural equations governing the hydrostatic equilibrium of a thermally relaxed, spherically-symmetric neutral lepton system with a density profile that can account for the dark matter distribution around spiral galaxies do not admit any physically reasonable solutions (namely, the temperature should be positive definite and monotonically decreasing with distance) if the lepton mass is \( \leq 17 \, \text{eV} \).

Attempts to explain the observed rotational curves in spiral galaxies in terms of 'invisible' halos made up of massive, neutral leptons have been encouraging (see Chau and Stone, 1985; and reference quoted therein). For the system M87 in particular, the agreement between theoretical and observational deductions on the mass distribution in the outer lying regions (\( \sim 100-400 \, \text{kpc} \)) is remarkable, considering the various constraints on the properties of the dark matter (cf. Chau and Lake, 1984). As long as answers to such fundamental questions as the exact mechanism of formation remain inclusive, it is fruitful to extract all the implications from a study of the static structure of such systems.

We present in this communication straightforward calculations that can apparently set a lower limit to the mass of the constituent particles (neutral leptons) in the 'invisible' halo. Specifically we will show that there exists no physically acceptable solution to the hydrostatic equilibrium structural equations if the lepton mass is less than \( \sim 17 \, \text{eV} \).

We assume that the invisible halo observationally deduced to be associated with spiral galaxies (cf. Rubin, 1983) is made up of one species of neutral leptons of mass \( m \) interacting only gravitationally with the visible (baryonic) matter. Furthermore, we assume the halo system is spherically-symmetric, and in hydrostatic and thermal equilibrium. The key to our calculations is that empirical fitting formulae to both the observed visible matter density distribution \( \rho_v \) and the required dark matter density distribution \( \rho_D \) are available for our Galaxy (a typical spiral) from the work of Bahcall et al. (1982). This then makes it easy to calculate the Newtonian gravitational force \( F_G \) (since previous work (Chau and Stone, 1985) has demonstrated that general relativistic effects are insignificant for such systems) in the structural equation for the lepton halo:

\[
\frac{dP}{dr} = F_G \rho_D ,
\]

where \( P \) is the pressure; \( F_G \), the gravitational force per unit mass; and \( r \), the usual radial variable.
$F_G$ can be split up into a spherically-symmetric part and a non-spherical symmetric part, the latter arising from the disk-component of $\rho_v$. Thus, we can write


(2)

Explicit expressions can be written for $F_G(SPH)$ and $F_G(DISK)$ from the known ones for $\rho_D$ and $\rho_v$, which we rewrite here for sake of completeness (see Chau and Stone, 1985, for more details):

$$\rho_D(r) = \rho_0 \left[ \left( \frac{a}{R_0} \right)^{1.1} + \left( \frac{r}{R_0} \right)^{1.1} + 0.045 \left( \frac{r}{R_0} \right)^{3.3} \right]^{-1},$$

(3)

with $\rho_0 = 1.124 \times 10^{-24} \text{g/cc}$,

$$a = 2 \text{kpc}, \quad R_0 = 8 \text{kpc};$$

and

$$\rho_v = \rho_v(DISK) + \rho_v(SPH). \quad \rho_v(DISK) = \rho_1 H \exp(-r/3.5 \text{kpc}),$$

(4)

with $\rho_1 = 6.53 \times 10^{-24} \text{g/cc}$ and $H = 7.03 \text{kpc}$, and

$$\rho_v(SPH) = 1.014 \rho_c \left( \frac{r}{1 \text{kpc}} \right)^{-1.5} \exp \left[ - \left( \frac{r}{1 \text{kpc}} \right)^3 \right] + \left. \begin{array}{l}
1.25 \left( \frac{r}{R_0} \right)^{6/8} \left\{ \exp \left[ -10.093 \left( \frac{r}{R_0} \right)^{1/4} + 10.093 \right] \right\}, \\
\frac{\rho_1}{800} \left( \frac{r}{R_0} \right)^{-7/8} \left\{ \exp \left[ -10.093 \left( \frac{r}{R_0} \right)^{1/4} + 10.093 \right] \right\} \times \left[ 1 - 0.08669 (r/R_0)^{1/4} \right], \quad r \geq 0.03R_0,
\end{array} \right\} \left\{ \begin{array}{l}
r < 0.03R_0, \\
r \geq 0.03R_0,
\end{array} \right\} \left\{ \begin{array}{l}
\right\},$$

(5)

with

$$\rho_c = 5.20 \times 10^{-17} \text{g/cc}.$$

With this, the expression for $F_G(SPH)$ is straightforward: namely,

$$F_G(SPH) = -\frac{GM(r)}{r^2},$$

(6)

where

$$\frac{\text{d}M(r)}{\text{d}r} = 4\pi r^2 (\rho_D + \rho_v(SPH));$$

(7)