SIMILARITY SOLUTIONS FOR PROPAGATION OF STRONG PLANE SHOCK WAVES IN AN OPTICALLY THIN ATMOSPHERE

G. DEB RAY
Dept of Mathematics, St Xavier's College, Calcutta, India
and
S. N. BANERJEE
Regional Computer Centre, Jadavpur University Campus, Calcutta, India

(Received 21 June, 1979)

Abstract. Similarity solutions, using Planck's diffusion approximation, for propagation of strong plane shock waves in an optically thin grey atmosphere of uniform density are obtained. Solutions for the ratio of specific heats 4/3 and several values of the radiation parameter are compared with those in the absence of any radiation. Excerpts from the results of integrations for different values of the ratio of specific heats are also given in Tables I-III.

1. Introduction

Propagation of plane shock waves in an optically thick gas has been studied in detail by several authors - e.g., Wang (1964), Koch (1965) and Helliwell (1969) and others. Self-similar solutions for spherical blast waves in air using Rosseland's diffusion approximation have been obtained by Elliott (1960) under the assumption of non-existence of heat flux at the centre of symmetry. A class of non-similar calculations for spherical blast waves in a transparent medium was carried out by Erickson and Olfe (1973). Conditions for the validity of whether a strong shock wave is optically thin have been investigated by Koch and Gross (1969) and Oppenheim (1972) using different radiation models. Further reference to shock waves with radiation effects may be had in the book by Zel'dovich and Raizer (1966).

In the present paper, similarity solutions for strong plane shock waves in a transparent grey atmosphere of uniform density but of very low uniform pressure have been obtained using Planck's diffusion approximation, unlike Wang (1964) and Helliwell (1969). These authors have also studied the piston-driven problem of a radiating gas in the optically thin limit, using a differential approximation for the equations of radiative transfer. Further, as the shock is very strong, the radiative flux across the transparent shock front becomes continuous. Besides, at the pre-explosion stage, it is supposed that emission of radiation is totally absent. This is justified as the gas is in a cold state. The medium may be conceived to have been impulsively disturbed by a planar explosion at the edge separating the medium from vacuum, as by the rupture of a membrane which initially separated the two media. The motion is headed by a
shock front. The results of integrations for the flow and thermodynamic variables are presented in the form of graphs, when the ratio of specific heats is taken as 4/3. The solutions should be valid at an initial stage of the disturbance, taking the radiation mean free path to be sufficiently large.

Applications of these type of solutions for a transparent gas may also be envisaged in laboratories in laser-induced shock tube experiments. Excerpts from the results of integrations for different values of the ratio of specific heats are given in Tables I-III.

2. Equations of Motion and Boundary Conditions

The equations of motion behind a plane shock are

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r} (\rho u) = 0, \]  

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \]  

\[ \frac{1}{(\gamma - 1)} \left[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right] - \frac{\gamma}{\rho (\gamma - 1)} \left[ \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial r} \right] + \frac{\partial F}{\partial r} = 0, \]  

where \( u \) is the material velocity, \( \rho \) the density, \( p \) the material pressure, and \( F \) the radiative flux all at a distance \( r \) from the plane of explosion at time \( t \); \( \gamma \) stands for the ratio of specific heats.

Also, assuming local thermodynamic equilibrium and taking Planck's diffusion approximation, we have

\[ \frac{\partial F}{\partial r} = 4K_\text{p} \sigma T^4, \]

where \( K_\text{p} \) is the Planck mean absorption coefficient, \( \sigma \) the Stefan–Boltzmann constant, and \( T \) the absolute temperature. As the gas is taken as ideal, the equation of state is given by

\[ p = \Gamma \rho T, \]

\( \Gamma \) being the gas constant.

We next take \( K_\text{p} \) as a power-law function of the density and temperature as

\[ K_\text{p} = \mu_0 \rho^\alpha T^\beta, \]

\( \mu_0, \alpha', \) and \( \beta' \) being constants.

The conditions across a strong shock are

\[ \frac{u_1}{V} = \frac{2}{\gamma + 1}, \]

\[ \frac{q_1}{q_0} = \frac{\gamma + 1}{\gamma - 1}, \]

\[ \frac{p_1}{q_0 V^2} = \frac{2}{\gamma + 1}, \]