STABILITY OF MAGNETO-ACOUSTIC WAVES IN A THERMALLY CONDUCTING COMPRESSIBLE FLUID

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Abstract. The stability of magneto-acoustic waves in an inviscid, perfectly conducting isothermal fluid, stratified under constant gravity and subjected to a horizontal magnetic field is investigated in the presence of thermal dissipation.

1. Introduction

It is widely accepted that magneto-acoustic waves play an important role in several phenomena associated with the terrestrial and solar atmospheres. A considerable effort has therefore gone into the study of these waves, especially from the viewpoint of their propagating characteristics. It is generally believed that the main source of heat input into the solar chromosphere and corona is the energy derived from shocks that develop out of acoustic waves originating in the solar convection zone (Schwarzschild, 1948). Other sources of energy input that would be present are photoionization by ultraviolet quanta and heating by corpuscular radiation that is generated in solar active regions. In order for the solar atmosphere to maintain a reasonably steady temperature, the rate of heating must be balanced by the rate of energy loss. While this may be true on the average, it is well known observationally that transient phenomena (such as solar flares and prominences) do occur in the chromosphere and corona. It is therefore desirable to enquire whether some aspects of these phenomena can be attributed to the thermal amplification of small disturbances in the solar atmosphere. The complete problem of investigating the waves excited in such atmospheres should take into account the full effects of the variation of physical variables characterizing the fluid. In practice, such an investigation becomes mathematically rather intractable when the various dissipative processes prevailing in these layers are included. Most of the work in this direction has been restricted to a homogeneous atmosphere or to the Boussinesq approximation.

Following the pioneering work of Rayleigh, which considered waves propagating vertically in an isothermal atmosphere, a number of attempts have been made to study the general problem of propagation of waves in a non-dissipative atmosphere with a varying physical environment. Yu (1965) studied the magneto-acoustic waves in an ideally conducting, non-dissipative isothermal fluid stratified under a constant vertical gravitational field and which is pervaded by a horizontal magnetic field varying in such a manner as to maintain a constant ratio between the magnetic pressure and the thermal pressure. Yu's work is restricted to the case when the fluid is
stable against convection. The assumption concerning the variation of the magnetic field with height renders the mathematical problem tractable by making the resulting differential equations have constant coefficients. It is then possible to obtain the dispersion relation in a polynomial form. Recently, Rudraiah et al. (1977) investigated the conditions for propagation and reflection of waves at various levels in such an isothermal fluid, with special attention to the horizontal and vertical components of the group velocity. Both these works are restricted to non-dissipative fluids. Singla and Talwar (1973) have obtained the dispersion relation by including the effect of heat loss mechanism. The effect of dissipative forces is also considered by Goldreich and Schubert (1967), Gilman (1970), Roberts and Stewartson (1977), Acheson (1978) and others. However, these workers have relied explicitly on the local dispersion relation with special reference to real roots only.

The present work may be considered as an extension of the work of Singla and Talwar to the case of overstable modes. We have attempted to make a complete analysis of all the modes that can be excited in such an isothermal fluid in the presence of thermal dissipation. Admittedly, the isothermal model which we have chosen is not too realistic as far as the terrestrial or solar atmosphere is concerned. Nevertheless, the study of such an idealized model enables us to gain an insight into the variety of modes that can be excited in more complicated systems which allow for a variation of the various physical quantities across the layer. The isothermal model, which is a special case of a polytropic fluid-layer, is a valuable first step in the study of unstable modes arising in compressible media. The investigation provides us with mathematically tractable equations which can be studied analytically and the dispersion relation can be reduced to a polynomial equation capable of analysis using the Routh–Hurwitz criteria.

2. The Dispersion Relation

We consider an ideally conducting, inviscid medium pervaded by a magnetic field. The governing hydromagnetic equations in the usual notation are

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\begin{align*}
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) &= -\nabla P + \rho \mathbf{g} + \frac{\mathbf{j} \times \mathbf{B}}{c}, \\
\rho \mathbf{v} \cdot \nabla \mathbf{v} &= 0, \\
\rho C_v \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) + P \nabla \cdot \mathbf{v} &= -\rho L, \\
P &= R\rho T, \\
\nabla \times \mathbf{H} &= \frac{4\pi}{c} \mathbf{j}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\
\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} &= 0.
\end{align*}
\]

We assume the gas constant \( R \), acceleration due to gravity \( \mathbf{g} \) and specific heat at constant volume \( C_v = R/(\gamma - 1) \) to be constants; \( \gamma \) is the ratio of specific heats, and