LINEAR ANALYSIS OF THE LIGHT CURVES
OF ECLIPSING VARIABLES

I. Analysis of the Fractional Loss of Light

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Abstract. In this paper, general model analysis of the form \( \sum c_i \phi_i(\theta) \) for the fractional loss of light \( f(\theta) \) exhibited by a close binary system will be considered in the sense of the least-squares criterion. A general recursive method will be established for constructing the normal equations for the most useful functions \( \phi_i(\theta) \), and an economical storage of the method on a digital computer, with its computational steps, will also be given. Moreover, a full recursive computational algorithm for the least-squares approximation will also be established. By means of this algorithm, all the solution vectors, the variance for different orders of fit and the corresponding variance–covariance matrix could be computed once and for all and, moreover, recursively. The economical storage of the algorithm and its computational steps will be given. Finally, some of the practical difficulties encountered in the application of the least-squares criterion will be analysed, and some techniques for detecting and controlling these difficulties are also given. Numerical examples on the Algol system using a Fourier cosine series of the form \( \sum c_i \cos [(j - 1)\pi \theta / \pi] \) will be given for illustration.

1. Introduction

A determination of the elements of eclipsing binary systems from an analysis of their light changes constitutes the basic task. The general framework of obtaining the elements of a given light curve are contained in the following three major problems:

1. the fitting of the fractional loss of light exhibited by the system in question;
2. the determination of the appropriate moments of the light curve;
3. the determination of the elements from the moments.

In this series we shall deal with each of these problems from analytical and numerical aspects of linear analysis.

In the present paper, we start with the first problem – that is, the fractional loss of light in the linear analysis.

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2. Statement of the Problem

Let
\[ f(\theta) = 1 - l(\theta) \]  
be the fractional loss of light exhibited by a close binary system at the phase angle \( \theta \) and \( \{ \theta_k \}, k = 1, 2, \ldots, m, \) be a sequence of data points assumed free from errors. Corresponding to each \( \theta_k \) we have, from visual or photographic observations, a number \( l_k \) of some function \( l(\theta) \) at \( \theta_k \) which generally will be in error. On the basis of the observations \( l_1, l_2, \ldots, l_m \) and Equation (2.1) we have the \( m \) values \( \check{f}_1, \check{f}_2, \ldots, \check{f}_m \) of some function \( f(\theta) \). These values are, of course, in error due to those of \( l_k \). We denote \( f(\theta_k) \), the true value at \( \theta_k \), by \( f_k \) and define
\[ \delta_k = f_k - \check{f}_k; \quad k = 1, 2, \ldots, m \]  
as the observational errors at different data points.

As we have said, the first problem in the analysis of the light curve of a given system is to approximate or fit the data \( \check{f}_k \) by some function \( \hat{P}(\theta) \) in such a way that \( \hat{P}(\theta) \) contains or represents most (if not all) the information about \( f(\theta) \) contained in the data and little (if any) of the errors.

This is accomplished in practice by selecting a function
\[ P(\theta) = P(\theta; c_1^{(n)}, c_2^{(n)}, \ldots, c_n^{(n)}), \]  
which depends on the parameters \( c_1^{(n)}, c_2^{(n)}, \ldots, c_n^{(n)} \). (The superscript \( n \) on \( c_i^{(n)} \) denotes the fact that they will generally depend on \( n \).) The function \( P(\theta) \) may be linear or non-linear in the parameters \( c_i^{(n)} \). Since our study is the linear-model analysis of the light curve, the function \( P \) will be selected as linear combinations of the parameters, so that \( P(\theta) \) will be of the form
\[ P(\theta) = \sum_{i=1}^{n} c_i^{(n)} \phi_i(\theta). \]  
The \( \{ \phi_i \} \) may, for example, be the set of monomials, exponentials, trigonometric polynomials, or indeed any arbitrary set of sufficiently defined functional values, provided only that they are \textit{linearly independent} of the \( m \) values of \( \theta \). Normally, \( n \) is small compared with the number, \( m \), of data points. In fact, the number \( n \) is unknown, but for the most appropriate approximation to a given data the number \( n \) should be
(1) large enough so that the information about \( f(\theta) \) in the data can well be represented by a proper choice of the parameters \( c_i^{(n)} \), while at the same time \( n \) should be
(2) too small to avoid the fitting of the observed data too closely in the sense that the errors in the observed data are retained in the approximation. Now the problem is to find the \textit{best} estimate \( \hat{P} \) of the function \( P \) - i.e., to determine particular values \( \hat{c}_1^{(n)}, \hat{c}_2^{(n)}, \ldots, \hat{c}_n^{(n)} \) for the parameters \( c_i^{(n)} \) to obtain the best approximation
\[ \hat{P} = P(\theta, \hat{c}_1^{(n)}, \hat{c}_2^{(n)}, \ldots, \hat{c}_n^{(n)}). \]