NEW TECHNIQUES OF LIGHT CURVE ANALYSIS:
APPLICATION TO V444 CYGNI

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Abstract. A new method of light curve analysis is introduced for systems containing one component possessing an extended atmosphere. Occultation or partial eclipses may be treated with or without the presence of transparency effects. Besides admitting an arbitrary degree of transparency to the eclipsing component, an arbitrary law of limb-darkening may also be assigned to the eclipsed star. The method is applied to the analysis of continuum, narrow band light curves of V444 Cygni obtained by Cherepashchuk and Khaliullin. Primary and secondary minima are examined separately with reasonable results.

1. Introduction

A new method for the analysis of the light curves of eclipsing variables is introduced. The effects of transparency as well as limb-darkening can be taken into account and the technique represents a novel approach to the study of the atmospheric eclipse. Such eclipses can be said to occur when at least one of the components possesses an extended atmosphere. There may be disagreement whether a particular star possesses this or an 'expanding envelope'. Such a distinction, however, need not concern us here. In both cases some radiation from the eclipsed star can be assumed scattered by the outer regions of the eclipsing star into the line of sight. The task here is to take this leaking radiation into account when solving for the elements by employing an adopted law of transparency.

The presence of an atmospheric eclipse may be inferred from the physical state of the stars – e.g., strong emission lines, or by light curve inspection. For example, if the minima are not of equal duration, even though radial velocity curves indicate circular orbits, then this would imply the presence of an envelope around one star that behaved differently as an occulting object than as a luminous object. Such is the case for V444 Cygni and the effects of the envelope are frequency dependent – i.e., due to electron scattering. For late-type stars such an envelope may not be easily detected. Here the opacity is both atomic and molecular, the depth of minima varying with the wavelength of the observation. Roach and Wood (1952) resolved the problem of whether the eclipse of ζ Aurigae could be explained by simple geometry. They established that the fraction of eclipse at a given phase is greater in the ultraviolet than in the blue. Such an effect cannot be accounted for by geometrical considerations.
There have been two types of systems for which extensive calculations have been made for atmospheric eclipse effects. These are systems which contain a Wolf-Rayet or a red supergiant, and of these V444 Cygni and ζ Aurigae can be said to form the respective prototypes. In this paper we apply the results to V444 Cygni while ζ Aurigae will be dealt with in a later paper.

2. The Atmospheric Eclipse

The literature treating the atmospheric eclipse has not been extensive. In an early work Kopal (1945) developed a formal method which determines the optical depth $\tau$ of a spherically-symmetric extended atmosphere. The method assumes the extinction coefficient to be constant with altitude, whilst the density varies exponentially. The treatment includes solutions for shallow eclipses where higher powers of $\tau$ can be neglected and general applications discussed. A subsequent refinement (Kopal, 1959, p. 239) permits the inclusion of deeper eclipses by optically thick atmospheres with steep density gradients.

Kopal and Shapley (1946), in an early study of V444 Cygni, determined the distribution of opacity in the semi-transparent envelope surrounding the Wolf-Rayet component. This was done by inverting an expression for the optical depth through the envelope as a function of $\delta$ in terms of the extinction coefficient. The inverted equation yields an expression for the extinction coefficient in terms of the optical depth and $\delta$.

Linnell (1958) developed a method somewhat similar to the earlier work of Kopal and Shapley. This approach differed in that it did not seek to determine a function for the optical depth directly from the light curve. Linnell's procedure assumed an analytic form and involved the construction of light loss tables for the initial determination of orbital parameters, including as one parameter a quantity specifying the extent of the atmosphere.

Cherepashchuk (1966a, b) takes a different approach to the solution of light curves of eclipsing binaries possessing one component with an extended atmosphere. This method involves the construction of an integral equation describing the brightness loss of the system in terms of geometrical and physical characteristics. This equation is then represented by a triangular system of linear algebraic equations solved numerically with adopted values for the geometrical and physical parameters. However, the method requires the use of rectified light curves and works only for eclipses of sufficiently strong maximum phase.

Following this Cherepashchuk (1974a, b) developed a refined method that entails a simultaneous solution of minima of the light curves. The basic equations are two Fredholm integral equations of the first kind which describe the light loss for the separate minima and another equation which expresses the normalization condition for the combined luminosity of the components in terms of the geometric parameters. The integral equations are solved numerically and the resulting solutions are used to determine the geometrical elements. If the condition $\cos i < r$ ($r$ is the radius of the