FIELD SOLUTION FOR A DIPOLE IN AN ANISOTROPIC MEDIUM CONTAINING TIME-VARYING IRREGULARITIES

S. S. DE
Centre of Advanced Study in Radio Physics and Electronics, University of Calcutta, India

and

A. C. SEN
Department of Physics, Bejoy Narayan Mahavidyalaya, Dist. Hoogly, West Bengal, India

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Abstract. Field solutions for a dipole in an anisotropic plasma medium are obtained in the presence of an infinitesimally small electric current source and time-varying irregularities through the use of inverse Fourier-transform technique. The derived solutions can be utilized for the evaluation of the field at an arbitrary distance and are also useful to lossy medium.

1. Introduction

The solutions of Maxwell's equations in an electromagnetic medium traversed by a magnetic field and in the presence of an infinitesimally small electric current source have been derived by many authors (Wait and Schlak, 1967; Singh and Gould, 1971; Alpert and Moiseyev, 1980; Pozar et al., 1985; Tsalamengas and Uzunoglu, 1985; De and Sen, 1987), which are of considerable importance for different applications.

Within the upper atmosphere as well as polar atmosphere, the time-varying irregularities give rise to time-varying magnetic field over and above the static field of the medium. This time-varying magnetic field further modifies the form of the dielectric tensor in the presence of collisions (Khan and De, 1970).

In this presentation, field solutions for a dipole in the case of an anisotropic medium appropriate to the ionospheric plasma in presence of time-varying irregularities along with the infinitesimally small electric current source are derived. The form of the solution so obtained may be utilized to calculate the fields at arbitrary distances from the source. The contribution towards the field due to time-varying irregularities may also be estimated. Moreover, the results are valid for complex elements and are thus applicable to lossy medium as well.

2. Mathematical Formulations

The Lorentz force equation for the stated medium can be written as

\[ \frac{\partial \mathbf{v}}{\partial t} = \frac{e}{m_e} \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) - \eta \mathbf{v}, \]

where \( \eta \) is the collision frequency.
Maxwell's equations with the source term is given by

$$\nabla \times \mathbf{H} = j \omega \epsilon_0 (\hat{\epsilon}) \mathbf{E} + \mathbf{J}. \quad (2)$$

The other Maxwell's equations have also been considered in the derivation. Where $\eta$ is the collision factor; $\hat{\epsilon}$, the dielectric tensor; $\mathbf{J}$, the source current; and the other symbols have their usual significance. $\hat{\epsilon}$ will be obtained from Equation (1) using appropriate relations (Khan and De, 1970).

As the time-varying irregularities within the ionosphere give rise to time-varying magnetic field over and above the static geomagnetic field, the effective magnetic field $\mathbf{B}$ can be written as

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 e^{j\omega t}, \quad (3)$$

where $\mathbf{B}_0$ is the Earth's magnetic field and $\mathbf{B}_1 e^{j\omega t}$ is due to time-varying irregularities.

Introducing dielectric polarisation $\mathbf{P} (= N\mathbf{e})$ and the relation (3), Equation (1) under Fourier transform through the aid of Faltung theorem, yields

$$e\mathbf{E} = \frac{m}{N_e} (j\omega \eta - \omega^2) \mathbf{P} - j\omega \mathbf{P} \times [\mathbf{B}_0 + \mathbf{B}_1 \delta(\omega - \omega_0)], \quad (4)$$

where $\delta$ denotes Dirac's delta function.

From Equation (4), the expression for the dielectric tensor can be derived as

$$\hat{\epsilon} = \left( \begin{array}{ccc} 1 - XX_1 & jXX_2 & 0 \\ -jXX_2 & 1 - XX_1 & 0 \\ 0 & 0 & 1 - (X/\omega^2) \end{array} \right) + \eta X \left( \begin{array}{ccc} -jX_3 & -X_4 & 0 \\ X_4 & -jX_3 & 0 \\ 0 & 0 & -j/\omega^3 \end{array} \right) + \frac{XX_1 \delta(\omega - \omega_0)}{\omega^2} \left( \begin{array}{ccc} -R_1 & jR_2 & R_3 - jR_4 \\ -jR_2 & -R_1 & R_5 + jR_6 \\ R_3 + jR_4 & R_5 - jR_6 & 0 \end{array} \right), \quad (5)$$

where

$$X = \omega_p^2, \quad X_1 = \frac{1}{\omega^2 - \omega_c^2}, \quad X_2 = \frac{i\omega_c}{\omega(\omega^2 - \omega_c^2)},$$

$$X_3 = \frac{\omega^2 + \omega_c^2}{\omega(\omega^2 - \omega_c^2)^2}, \quad X_4 = \frac{2\omega_c}{(\omega^2 - \omega_c^2)^2},$$

$$R_1 = \frac{2\omega^2 \omega_c \omega_{ex}}{\omega^2 - \omega_c^2}, \quad R_2 = \frac{\omega(\omega^2 + \omega_c^2) \omega_{ex}}{\omega^2 - \omega_c^2}, \quad R_3 = \omega_c \omega_{ex}.$$