Abstract. A non spherically-symmetric monoenergetic-point-source solution of the steady-state equation of transport for cosmic-rays in the interplanetary region, in which monoenergetic particles are released isotropically and continuously from a fixed heliocentric position is derived by a Laplace transform method. The solution is for a spherically-symmetric model of the propagating region incorporating anisotropic diffusion, with a diffusion tensor symmetric about the radial direction, and the solar wind velocity is radial and of constant speed $V$. The spherically-symmetric monoenergetic-source solution of Webb and Gleeson (1973) and of Toptygin (1973) is regained from the spherically-symmetric component of the point-source solution.

1. Introduction

The equation of transport for cosmic-rays in the interplanetary region, including convection, diffusion and energy change processes was initially obtained by Parker (1965) and later by Gleeson and Axford (1967), and by Dolginov and Toptygin (1967, 1968).

Steady-state spherically-symmetric analytic solutions of the equation have been given by Parker (1965, 1966), Dolginov and Toptygin (1967, 1968), Fisk and Axford (1969), Toptygin (1973), Webb and Gleeson (1973, 1974), Gleeson and Webb (1975) and Webb (1976a, b, c), and numerical solutions have been obtained following Fisk (1969) and Urch (1971). In addition two useful approximate analytic solutions have been obtained by Gleeson and Axford (1968a, b).

Probably the most fundamental analytic solution is the monoenergetic-source or Green's function solution given by Webb and Gleeson (1973), without proof, and also by Toptygin (1973) who derived it by a Laplace transform technique. An alternative derivation of this solution has been given by Webb (1976a, c) using Lie theory. In the above solution monoenergetic particles are released isotropically and at a steady rate from a spherical shell at a fixed heliocentric radius, and the resultant redistribution of particles in momentum $p$ is determined as a function of heliocentric radius $r$. The solution was obtained for a model of the interplanetary region in which the effective radial diffusion coefficient $\kappa = \kappa_0(p)r^b$, with $b$ a constant, $\kappa_0(p)$ an arbitrary function of $p$, and the solar-wind velocity was assumed radial and of constant speed $V$.

In this paper, a monoenergetic point-source solution of the equation of transport is derived for a spherically-symmetric model of the interplanetary region incorporating anisotropic diffusion. The monoenergetic particles are released isotropically and at a
steady rate from a fixed point within the interplanetary region. The solution for the distribution function $F_0(r, \theta, \phi, p)$ is a function of particle momentum $p$, and heliocentric position $(r, \theta, \phi)$ where $(r, \theta, \phi)$ are spherical polar coordinates centred on the Sun with polar axis along the Sun's rotation axis. This is in contrast to the spherically-symmetric solution of Webb and Gleeson (1973) and of Toptygin (1973) in which monoenergetic particles were released isotropically from a spherical shell at a fixed heliocentric radius $r_0$.

The interplanetary magnetic field is assumed to be radial; with a diffusion tensor symmetric about the magnetic field, and the solar-wind velocity is assumed radial and of constant speed $V$.

In Section 2, the steady-state equation of transport is expressed in a separable form, with separation variables $(x, t, \mu, \phi)$ where $x$ is a function of $r$ and $p$, $t$ is a function of $p$ and $\mu = \cos \theta$. The monoenergetic point-source solution is then derived in Section 3, by a Laplace transform method. By superposition of monoenergetic point-source solutions with sources on a fixed spherical shell, the spherically-symmetric monoenergetic-source solution is regained.

Finally in Section 4, some comments are made about the scope and applicability of monoenergetic-source solutions, and their basic role in cosmic-ray studies from a physical and mathematical point of view.

2. Separable Forms of the Cosmic-Ray Equation of Transport

With a source of monoenergetic particles of momentum $p_0$ released at a heliocentric position $r_0$, the steady-state equation of transport for cosmic-rays in the interplanetary region is (Parker, 1965; Gleeson and Axford, 1967; Jokipii and Parker, 1970; Dolginov and Toptygin, 1967, 1968)

$$\nabla \cdot (VF_0 - K \cdot \nabla F_0) - \frac{1}{3p^2} \nabla \cdot V \frac{\partial}{\partial p} (p^3 F_0) = \frac{N_0 \delta(r - r_0) \delta(p - p_0)}{4\pi p_0^2}, \quad (2.1)$$

where $F_0(r, p)$ is the mean distribution function with respect to momentum $p$ at heliocentric position $r$; $V$ is the solar wind velocity; $K$ the diffusion tensor; and the right-hand side of Equation (2.1) represents the source with $N_0$ the rate at which particles are injected.

The interplanetary magnetic field is assumed to be radial, with diffusion coefficients $\kappa_\parallel$ along and $\kappa_\perp$ perpendicular to the magnetic field. The diffusion coefficients are assumed to have the form

$$\kappa_\parallel = \kappa_0(p) r^b, \quad \kappa_\perp/\kappa_\parallel = e = \text{constant}. \quad (2.2)$$

By introducing new independent variables $(x, t, \mu, \phi)$ and the variable $T$ where

$$x = \begin{cases} 
2(rp^{3/2})^{(1-b)/2}\left|1 - b\right| & \text{if } b \neq 1, \\
-\frac{1}{2} \ln \left(2rp^3\right) & \text{if } b = 1,
\end{cases} \quad (2.3)$$