ON A UNIFORMLY ROTATING MAGNETIC POLYTROPE

II. The Equilibrium Structure with a Large Toroidal Magnetic Field

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Abstract. The equilibrium structure of a uniformly rotating magnetic polytrope in the presence of a large toroidal field has been determined for polytropic index \( n = 1.0, 1.5, 2.0 \) and 3.0. It has been found that the contribution of higher-order nonspherical terms in the determination of equilibrium structure becomes significant for large magnetic field.

1. Introduction

In a recent paper Das and Tandon (1977), have studied the effect of large general magnetic field on the equilibrium structure and oscillation of a uniformly rotating polytrope. It was assumed by them that the contribution of nonspherical terms of order \( h^2 \) or higher on the equilibrium structure is negligible in comparison to the spherically-symmetric contributions which were taken to all order in \( h \) (\( h \) is proportional to the ratio of magnetic to gravitational energy.) In view of the nonavailability of the exact solution for a polytrope with a prevalent general magnetic field and the mathematical complexity of the problem, the contribution of the higher-order nonspherical terms in the determination of the structural parameters could not be analysed. However, it is possible to take account of the effect of higher-order nonspherical contributions to the equilibrium structure for a purely toroidal magnetic field inside the polytrope. Earlier the first order contribution of \( h \) to the equilibrium structure of a nonrotating magnetic polytrope with a toroidal field has been considered by Roxburgh and Durney (1967), Sinha (1968), and Sood and Trehan (1972). Recently Miketinac (1973) has extended their analysis to greater accuracy using Stoeckly’s (1965) exact numerical method for the case of a rapidly rotating polytrope. This latter method, though exact, is, however, still complicated to handle and time-consuming on an average computer.

As the stars (Main Sequence) in general have rotation and magnetic field, we, therefore, in the present paper shall study the equilibrium structure of a uniformly rotating star with large toroidal field, by developing a second-order perturbation theory to incorporate the effect of sufficiently large magnetic field, and to determine the contribution of the higher-order nonspherical terms. The results thus obtained have been compared with those obtained by extending the analysis of Das and Tandon (1977) for the purely toroidal field. It has been found that for a weak field, the result obtained from this latter technique agrees well with those of the former. However,
for a rather large magnetic field, the effect of nonspherical terms becomes significant enough and the former technique is preferable for determining the equilibrium structure of a rotating magnetic polytrope.

2. Equilibrium Structure

2A. Second-order Perturbation Theory

The equilibrium structure of a magneto-rotating polytrope with a general magnetic field and rotation has been studied earlier by Das and Tandon (1976). Following their analysis, the modified Lane-Emden equation, determining the structure of a uniformly rotating polytrope with a toroidal magnetic field $H\phi = k\omega_0$, can be written as

$$\nabla^2 \Theta = -\Theta^n + h[A - \frac{1}{2}\nabla^2(\omega^2\Theta^n)],$$

where

$$h = \frac{k^2}{4\pi^2 G}$$

and

$$A = \Omega^2/2\pi G\omega_0 h.$$ 

In the foregoing equation $h$ and $A$ are the quantities proportional to the ratio of magnetic to gravitational energy and rotational to magnetic energy respectively and $k$ is related to strength of the magnetic field. Also $\omega_0$ denote the central density and $\Omega$ the angular velocity of the polytrope.

For sufficiently large magnetic field inside the polytrope, we write for the density function $\Theta(\xi, \mu)$ a solution of the form (Das and Tandon, 1977)

$$\Theta(\xi, \mu) = \Theta(\xi) + \sum_m h^m f_m(\xi) + \sum_m h^m \sum_i (\delta_{i, m} A_i \psi_{2m}(\xi) P_{2m}(\mu) +$$

$$+ \delta_{i+1, m} B_i \psi_{2m-2}(\xi) P_{2m-2}(\mu) +$$

$$+ \delta_{i+2, m} B_i \psi_{2m-2}(\xi) P_{2m-2}(\mu) + \cdots),$$

where $\Theta(\xi)$ is the Lane-Emden function and $\delta_{i, j}$ and $P_i(\mu)$ are the Kronecker delta and the Legendre polynomial, respectively.

Neglecting the terms of order higher than $h^2$, Equation (3) can be rewritten as

$$\Theta(\xi, \mu) = \Theta(\xi) + h[f_0(\xi) + \psi_2(\xi) \psi_2(\mu)] +$$

$$+ h^2[f_2(\xi) + F_2(\xi) P_2(\mu) + F_4(\xi) P_4(\mu)].$$

By substituting Equation (4) into (1) and on equating the coefficient of various order of Legendre polynomials, we get

$$\frac{d^2 \Theta}{d\xi^2} + \frac{2}{\xi} \frac{d \Theta}{d \xi} = -\Theta^n,$$

$$\frac{d^2 \phi_0}{d\xi^2} + \frac{2}{\xi} \frac{d \phi_0}{d \xi} = -n\phi_0^{n-1} f_0 + \phi_{0,1}(\xi),$$

$$\frac{d^2 \psi_2}{d\xi^2} + \frac{2}{\xi} \frac{d \psi_2}{d \xi} - \frac{6}{\xi^2} \psi_2 = -n\psi_2^{n-1}\psi_2 + \phi_{2,1}(\xi),$$

where $\omega_0$ denote the central density and $\Omega$ the angular velocity of the polytrope.